

PROGRESS/ ACTIVITY REPORT

Name of Grantee: Dr. Michael Oluniyi OGUNDIRAN

Host: Prof. Luigi ACCARDI

Host Institution: Volterra Centre, University of Rome, Tor Vergata.

Title: Stochastic limit of quantum theory.

Quantum stochastic calculus of Hudson-Parthasarathy(H-P) formulations driven by quantum noises; annihilation, creation and number operators had been generalized in various ways. The quantum stochastic differential equation from this calculus gave a generalization of classical Ito stochastic differential equation to non-commutative setting. In recent times, the White Noise Hamiltonian Equation (WNHE) was shown to have the previous calculi as special cases.

The research activity during my visit started with the review of the state of art of the subject matter, precisely, the stochastic limit of quantum theory. It was first noted that white noise approach to stochastic analysis gave a unification in the study of both classical and quantum stochastic differential equations. It was also noted that WNHE gives a non-linear extension of stochastic differential equations to the study of fields instead of processes. Finally, for more applications to be possible, the study of WNHE for unbounded coefficients cases was yet open.

The study of quantum stochastic calculus from the point view of algebraic quantum problem enable applications of the quantum stochastic calculus to quantum open problems, quantum electrodynamics, etc. in a prescribed operator algebra. The activity of the first one week was on the study of algebraic quantum probability and fundamentals of stochastic limit of quantum theory.

In order to solve problems involving stochastic differential equations (classical or quantum), the first challenge to circumvent is the estimation of the integral representations of the equation. In the work, we continued our activity by establishing suitable integral estimates for the existence of solution for WNHE with unbounded coefficients. To do this some new tools were employed because of the nature of our domain of definition for the operators involved. The maximal algebraic domain has an intrinsic operator property necessary for physical application of WNHE. At the end of the fourth week, mastering the new technique and proving the integral estimates were fully achieved.

There are different cases of WNHE suitable for various physical applications. The study and establishment of basic results needed for the proof of existence results for these cases was the objective of the last two weeks. During these weeks there was a short interaction with another visiting expert in quantum stochastic analysis from Korea, who was on a visit to the centre for one week. The direction of his study of white noise analysis is via some white noise operators in suitable locally convex spaces. A blend and unification of the two approaches

will be our future research collaboration especially for applications to quantum stochastic control.

In summary, my collaborative research work with my host, Prof. Accardi had not only yielded the results to be published, but had opened to me and invariably other colleagues in my country a new research direction of quantum stochastic analysis. Prof. Accardi and I had also decided and started working on organising school on quantum probability and applications in Nigeria. This will facilitate research in infinite dimensional analysis and its applications for Mathematics and Physics Faculties/Researchers and graduate students in the country.

The only challenge I had was the delay in visa processing which made me incur extra cost on air fare due to flight re-schedule and a short of few days in my visit.

In conclusion, I want to express my sincere appreciation to International Mathematical Union (IMU) and Committee for Developing Countries (CDC) for African SIMON's fellowship. It has actually been of great benefit to me and invariably to other researchers (of same interest) in my home country who will benefit from me through interactions.

Thank you

Ogundiran, Michael Oluniyi (Ph.D.)