CIMPA Research School Final Report

Title of School : Combinatorial Commutative Algebra

Dates : March 05 – 14, 2018

Location : Lahore (Pakistan)

I. Summary

The CIMPA School on Combinatorial Commutative Algebra was held at Department of Mathematics, COMSATS Institute of Information Technology(CIIT), Lahore Campus on March 05-14, 2018. A total of 78 researchers including 6 speakers, 11 international students (India (3), Iran (1), Indonesia (2), China (1), Morocco (1), Malaysia (1), Vietnam (1) and Italy (1)) and 60 national students have participated in the event. The opening ceremony of the event was held on March 05, 2018 in which Prof. Dr. Arshad Saleem Bhatti, Dean of Sciences CIIT Islamabad, Prof. Dr. Jorge Mozo Fern´andez, representing CIMPA, Laura Mayoral Aguilera, Spanish Consul, representing the Spanish Embassy in Pakistan, Prof. Dr. A. D. Raza Choudhary, ex Director General Abdus Salam School of Mathematical Sciences, GCU Lahore-Pakistan were guests of honor. The school lectures were held at seminar room of science block. The seminar room has 125 fixed chairs, two big white board, a multimedia projector, sound system and fully air-condition system. Each day was divided in two sessions. In each morning session there used to be three lectures of one hour each while evening sessions were dedicated to tutorials and short presentations by the participants. Ten students have presented their research while many students had actively participated in the tutorial sessions. There were two excursions including visits to the Army Museum, Badshahi Mosjid and Wagha Border (Pakistan India Border). Out of 6 speakers, 3 had stayed within the university campus while remaining speakers had stayed at a nearby hotel along with other international participants. There were 12 Pakistani students who came from outside Lahore to participate in the school. The CIIT Lahore campus management had provided them full residence. The transport to all students from Lahore and to those staying in nearby hotels was also provided by CIIT Lahore Campus.

II. Scientific Content

The CIMPA School on Combinatorial Commutative Algebra consisted of six courses with a number of lectures for each as per following detail:

Course 1. Volkmar Welker (Germany): Orthogonal Representation of Graphs, Combinatorics and Algebra.

Motivated by questions in algebra and combinatorics we study two ideals associated to a simple graph G on n vertices:

- the Lovasz-Saks-Schrijver ideal defining the orthogonal representations in $\mathbb{R}^d$ of the graph complementary to G and
- the ideal of the $(d+1)$ −minors of a generic symmetric $n \times n$ −matrices with 0s in positions prescribed by the graph G.

We show that these two ideals are closely related and that algebraic properties such as being radical, prime or a complete intersection/having expected height transfer from the Lovasz-Saks-Schrijver ideal of orthogonal representations in $\mathbb{R}^d$ to the ideal of $(d+1)$ −minors. For $d = 1$ the Lovasz-Saks-Schrijver ideal of G is known as the edge ideal of G and for $d = 2$ and G bipartite it is up to coordinate transformation the binomial edge ideal of G.

An orthogonal representation of a graph G in $\mathbb{R}^d$ is a map ϕ from the set of vertices of G to $\mathbb{R}^d$ such that vertices not connected by an edge in G are mapped to orthogonal vectors. The Lovasz-Saks-Schrijver ideal of a graph G is then generated by the equations exressing the orthogonality relations of an orthognal representation of the complement of G. Orthogonal representations were first studied by Lovasz in connection to graph entropy. In work with Saks and Schrijver he also studied the geometry of the variety of orthogonal graph representations. The latter is the reason for naming the ideals Lovasz-Saks-Schrijver ideals.

For Lovasz-Saks-Schrijver ideals we link radicality, complete intersection and primality to combinatorial properties of G and show that they always hold for d large enough. For specific classes of graph, such a forests, we can give a complete picture and classify the radical, prime and complete intersection Lovasz-Saks-Schrijver ideals.

Course 2. Tim Römer (Germany): Lattice Polytopes.

In these lectures we discuss lattice polytopes and associated rings. The latter are toric algebras. The interplay between combinatorial properties of the polytopes and the corresponding algebraic ones of the algebras is an active area of research in combinatorial commutative algebra. We discuss examples of such results and useful methods to study these

objects. Moreover, some lattice polytopes of interest are considered to which we apply the theory.

Course 3. Sara Saeedi Madani (Iran): Binomial Edge Ideals

In these lectures we introduce a class of binomial ideals attached to graphs, called "binomial edge ideals", and we study various algebraic properties and invariants of them. We try to understand those properties and invariants via the combinatorial properties of the underlying graph. Finally, we present some open problems in this area.

Course 4. Santiago Zarzuela Armengou (Spain): Computing Local Cohomology

Local cohomology is difficult to compute explicitly. One can reduce to a simple set-up as in the Hochster's formula for the local cohomology of a Stanley-Reisner ring. The poset defined by the primary decomposition of the defining ideal provides the needed combinatorial information. These ideas can be extended in several directions, always with the above poset as the combinatorial object where to look. In these lectures we shall review some of these constructions, which often involve the explicit computation of the derived functors of the direct and inverse limits over a finite poset.

Course 5. Nguyen Dang Hop (Vietnam): Powers and Symbolic powers of ideals

The study of powers and symbolic powers of ideals in a ring is a classical topic in commutative algebra. Let R be a polynomial ring over a field with the standard grading, and I a homogeneous ideal of R. There are two aspects in the study of powers and symbolic powers of I. Firstly, there is the asymptotic study, which focus on the eventual behavior of large enough powers of I. We usually expect uniform behavior of the large enough powers. But there is also the wholesale study, in which we ask about the common feature of all the powers. An example in the wholesale study is: Characterize the sequence $(depth\, R/I^n$, where n ranges over the natural numbers? The second aspect is less well-understood, as usually we don't expect that the small powers of I to behave in a simple manner. We will discuss both asymptotic and wholesale aspects of powers and symbolic powers. In particular, we will focus on the depth and the Castelnuovo-Mumford regularity of these powers. We will use tools like local cohomology, simplicial homology, usually only in simple ways, to answer some algebraic questions on powers (and symbolic powers) of ideals.

Course 6. Rashid Zaare-Nahandi (Iran): Ideals with Linear Quotients and the Simon Conjecture

Let I be a monomial ideal in the ring of polynomials $k[x_1, \dots, x_n]$ generated by $u_1, .., u_m$. It is called an ideal with linear quotients if the colon ideal $(u_1, .., u_i) : (u_{i+1})$ is generated by linear forms, for each $1 \leq i < m$. A hypergraph G is called a clutter if cardinality of all hyper edges are the same. For each clutter an edge ideal $I(G)$ generated by equidimensional monomials is corresponded. In the first lecture, we introduce the notion of a simplicial element and chordality in clutters. In the second lecture, properties of ideals with linear quotients will be discussed and a theorem will be proved that any equidimensional ideal with linear quotients can be obtained by a simplicial order in a complete clutter. In the third lecture, a conjecture introduced by Simon on extedable shellablity property of clutters and its relation to simplicial orders will be discussed. This work is joint with Mina Bigdeli, Jurgen Herzog and Ali Akbar Yazdan Pour.

Following 10 students have presented their research work during the school.

1) Carla Mascia (Italy)
2) Azeem Khadam(Pakistan)
3) Asma Khalid(Pakistan)
4) Tahira Majeed(Pakistan)
5) Naqeeb-ur-Rehman(Pakistan)
6) Rabia Nazir(Pakistan)
7) Muhammad Imran Bhat(India)
8) Fatimah Abdul Razzak (Indonesia)
9) Rezwan-ul-Shaban(India)
10) Hafizullah(Pakistan)

III. Participants

This school turned to be very successful in terms of participation, where out of total of 61 participants, 11 foreign and 50 Pakistani participants joined the school. Initially 19 participants were approved by CIMPA but at the end only 11 managed to turn up. Out of these only 8 were partially or fully funded by CIMPA to cover either of their travel, food and lodging cost. The foreign participants; 8 male and 3 female were from Iran (1), India (3), Indonesia (2), Italy (1), Congo (1), China (1), Vietnam (1) and Malaysia (1). The names of participants are as under:

1. Volkmar Welker Philipps-Universität Marburg, Fachbereich Mathematik

22. Afshan Adil	
23. Aqsa	COMSATS Institute of Information Technology, Islamabad
24. Aqsa Bashir	ASSMS, GC University, Lahore, Pakistan
25. Asia Rauf	Govt. College Women University Faisalabad, Pakistan
26. Asma Khalid	ASSMS, GC University, Lahore, Pakistan
27. Azeem Khadam	ASSMS, GC University, Lahore, Pakistan
28. Faraha Ashraf	ASSMS, GC University, Lahore, Pakistan
29. Fareeha Ambar	Lahore University of Management Sciences (LUMS), Lahore
30. Ghazanfar Abbas	ASSMS, GC University, Lahore, Pakistan
31. Hafiz Ullah	CIIT Islamabad, Pakistan
32. Hafiza Mehreen Zafar	Lahore College for Women University, Jhang Campus
33. Haseeb Wali	CIIT Islamabad, Pakistan
34. IMRAN ANWAR	ASSMS, GC University, Lahore, Pakistan
35. Maria Naseem	University of Central Punjab, Lahore, Pakistan
36. Maria Farooq	Lahore University of Management Sciences (LUMS), Lahore
37. Mariam Imtiaz	University of Engineering &Technology, Lahore, Pakistan
38. Muhammad Zahid	ASSMS, GC University, Lahore, Pakistan
39. Nadia Shoukat	ASSMS, GC University, Lahore, Pakistan
40. Nimra Javed	ASSMS, GC University, Lahore, Pakistan
41. Rabia Nazir	Govt. College University Faisalabad, Pakistan
42. Rafiah Zafar	Lahore College for Women University Jhang Campus
43. Raza Ali	ASSMS, GC University, Lahore, Pakistan
44. Rehana Ashraf	Lahore College for Women University, Jhang Campus
45. Rida Irfan	COMSATS Institute of Information Technology, Sahiwal
46. Rizwan Jahangir	ASSMS, GC University, Lahore, Pakistan
47. Sadia Akhtar	Lahore College for Women University, Jhang Campus,
48. Sajjad Khan	CIIT Islamabad, Pakistan
49. Sidra Razzaq	Lahore College for Women University, Jhang Campus,
50. Syed Fazal Abbas Shah	ASSMS, GC University, Lahore, Pakistan

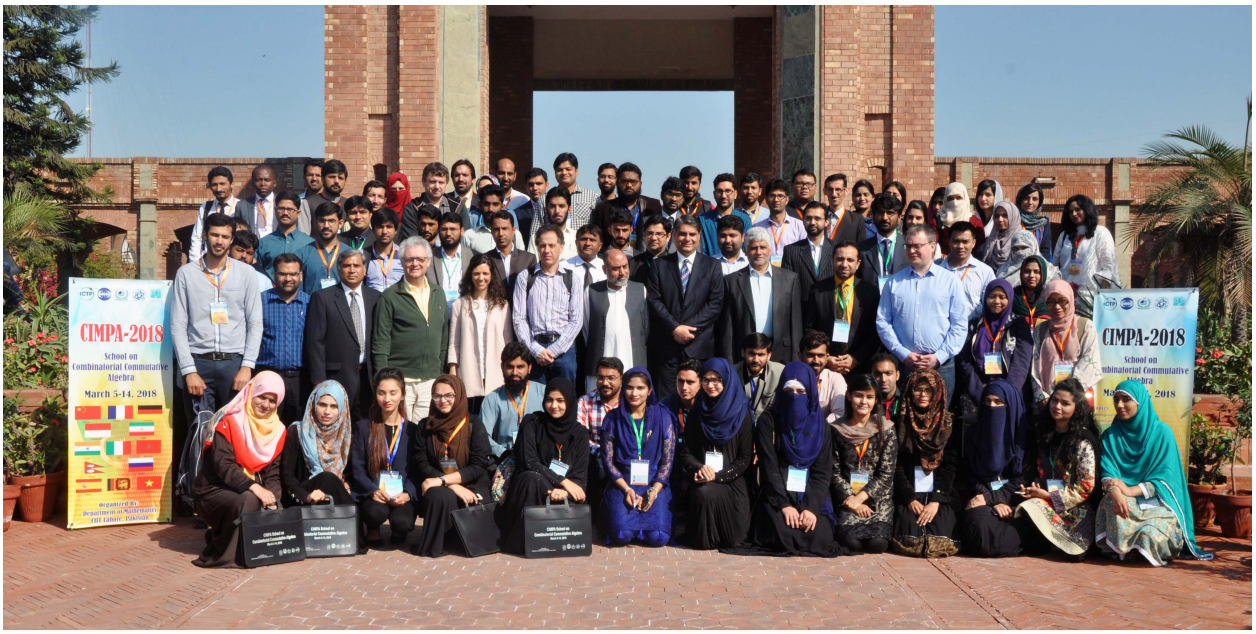
51. Tahira Majeed	COMSATS Institute of Information Technology, Lahore
52. Tusif Ahmed Malik	ASSMS, GC University, Lahore, Pakistan
53. Zahid Iqbal	National University of Science and Technology, Islamabad
54. Zainab Ali	Lahore University of Management Sciences (LUMS), Lahore
55. Zohaib Nadeem Sheikh	ASSMS, GC University, Lahore, Pakistan
56. Zunaira Kosar	ASSMS, GC University, Lahore, Pakistan
57. Asif Allah Ditta	COMSATS Institute of Information Technology, Lahore
58. Dilwar Ali	COMSATS Institute of Information Technology, Lahore
59. Wakeel Ahmad	COMSATS Institute of Information Technology, Lahore
60. Saad Ahmad	COMSATS Institute of Information Technology, Lahore
61. Aqsa Farooq	COMSATS Institute of Information Technology, Lahore
62. Iqra Farman	COMSATS Institute of Information Technology, Lahore
63. Muhammad Awais	COMSATS Institute of Information Technology, Lahore
64. Sabir Hussain	COMSATS Institute of Information Technology, Lahore
65. M. Fouz Farooq	COMSATS Institute of Information Technology, Lahore
66. Iram Saleem	COMSATS Institute of Information Technology, Lahore
67. Hina Javaid	COMSATS Institute of Information Technology, Lahore
68. Haseeb Ahmad	COMSATS Institute of Information Technology, Lahore
69. Ali	COMSATS Institute of Information Technology, Lahore
70. Arhum Maqbool	COMSATS Institute of Information Technology, Lahore
71. Ammar Mujahid	COMSATS Institute of Information Technology, Lahore
72. Iqra Siddique	COMSATS Institute of Information Technology, Lahore
73. Ayesha	COMSATS Institute of Information Technology, Lahore
74. Malik Ali Raza	COMSATS Institute of Information Technology, Lahore
75. Saliha Manzoor	COMSATS Institute of Information Technology, Lahore
76. Zahra Manzoor	COMSATS Institute of Information Technology, Lahore
77. Asma	COMSATS Institute of Information Technology, Lahore
78. Azmat	COMSATS Institute of Information Technology, Lahore

IV. Financial Report

Sr. #	Funds	Granted (in Euros)	Spending (in Euros)
1.	CIMPA	10000	9423
2.	ICTP	1000	1000
3.	IMU-CDC	1500	1500
4.	COMSATS IIT, Pakistan	5000	4821
5.	HEC Pakistan	2896	2896
Total		20396	19640

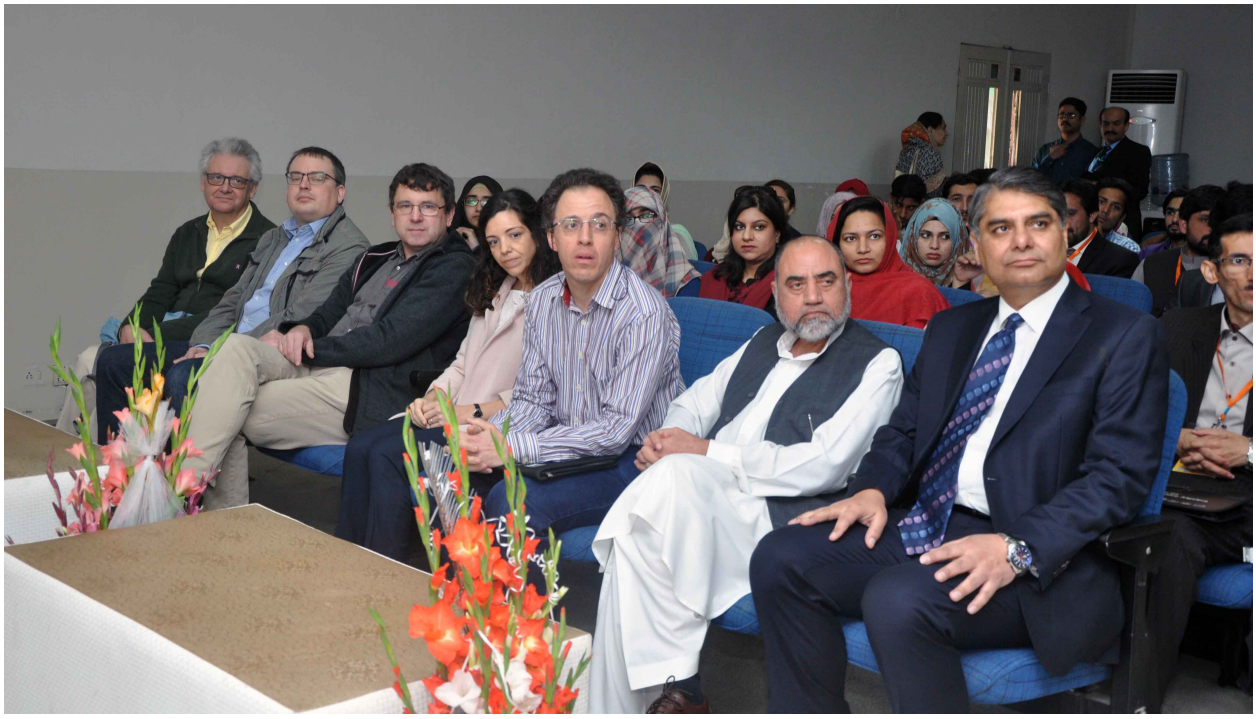
Table 1: Summary of funds granted and spending

V. Pictures



Group Photo

Cimpa Presentative Prof. Dr. Jorge Mozo Fern´andez addressing the audience at the opening ceremony

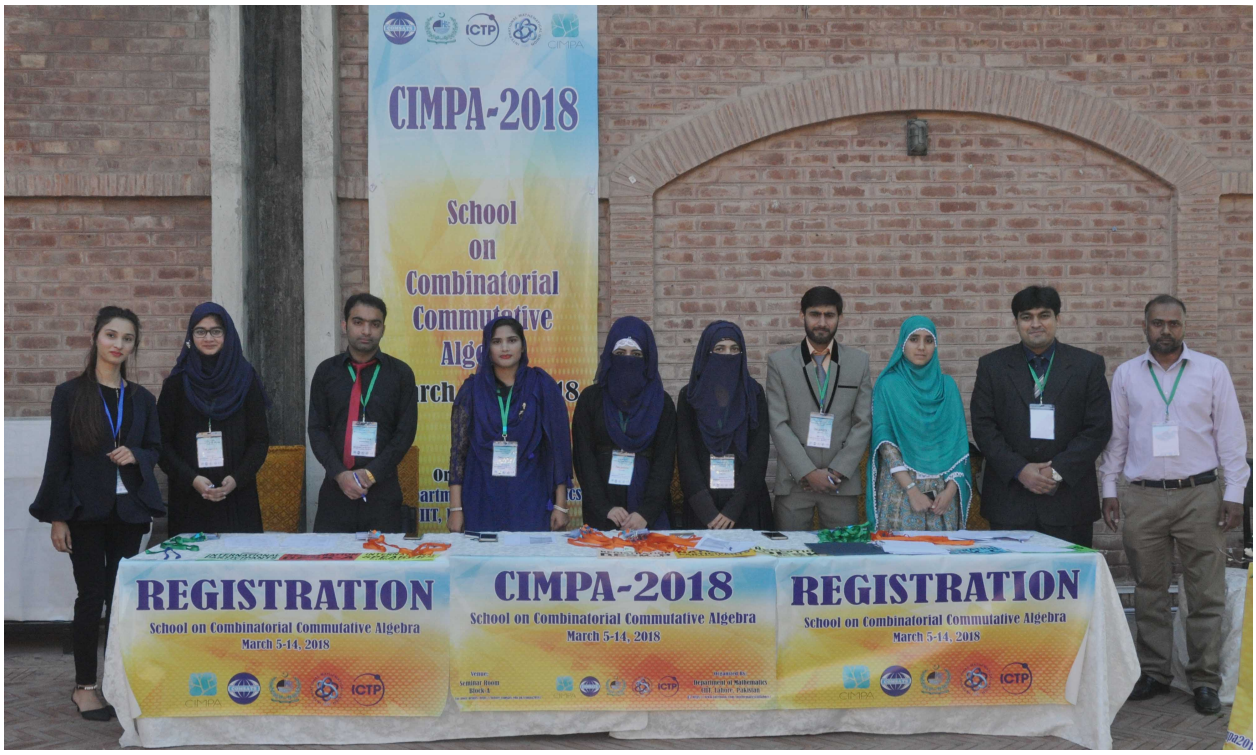


From left to right: Prof. Santiago Zarzuela Armengou (Spain), Prof. Tim Romer (Germany), Prof. Volkmar Welker, (Germany), Ms. Laura Mayoral Aguilera(Spanish Consul), Prof. Jorge Mozo Fernández (Spain), Prof. A. D. Raza Choudhary (Ex DG ASSMS), Prof. A. S. Bhati (Dean of Sciences, CIIT) at the opening ceremony

Ex DG ASSMS Prof. Dr. A. D. Raza Choudhar addressing the audience

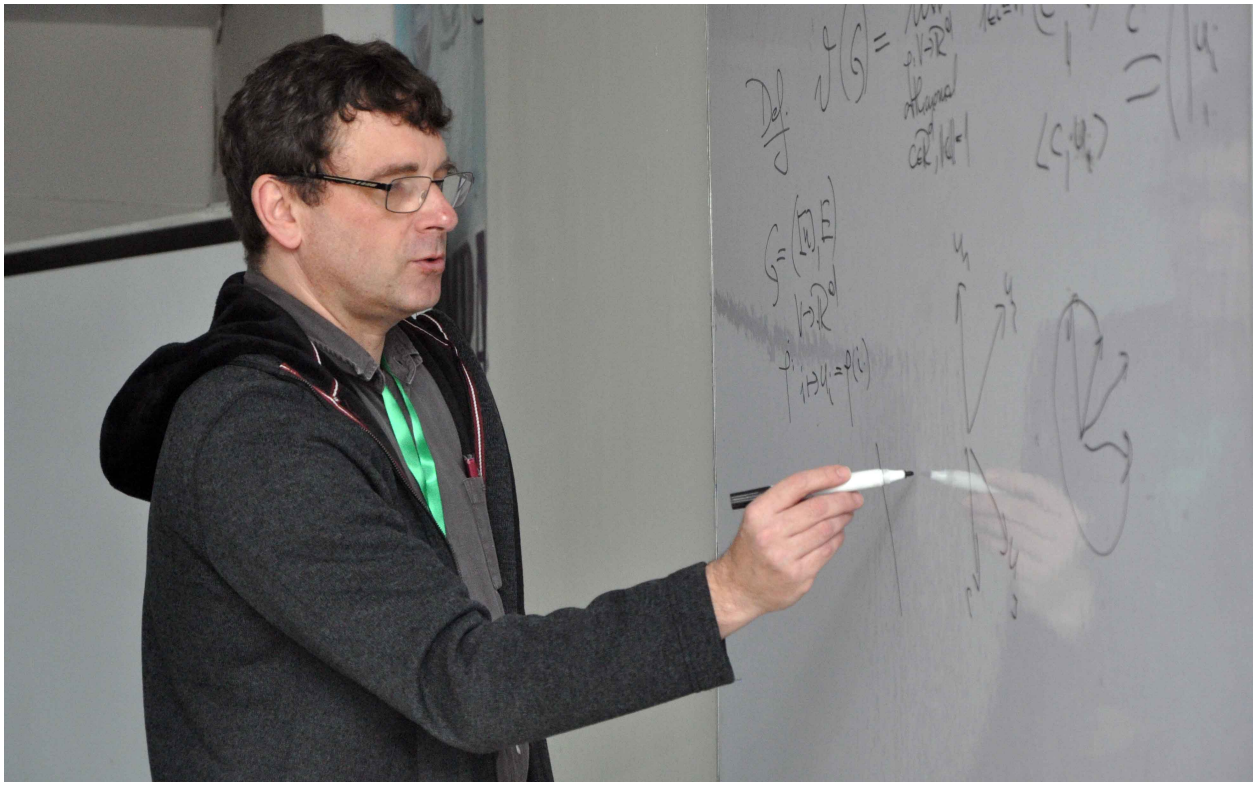
International participants at the opening session

Registration desks



Registration team


ICTP
CIMPA
CIMPA-2018
School on
Combinatorial Commutative
Algebra



Prof. Volkmar Welker (Germany) opening lecture