Emil Artin International Conference

Dedicated to the 120th Anniversary of Emil Artin
(03.07.1898-20.12.1962)

Yerevan, the Republic of Armenia,
May 27-June 2, 2018.

ABSTRACTS

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Yerevan 2018

EMS Conference

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Dedicated to the 120th Anniversary of Emil Artin (03.07.1898-20.12.1962)

Yerevan, the Republic of Armenia, May 27-June 2, 2018.

The conference is organized by:

- Armenian Mathematical Union
- Yerevan State University
- American University of Armenia
- Committee of Emil Artin Junior Prize in Mathematics
- Institute of Mathematics of National Academy of Sciences of Armenia
- University of Bergen
- Steklov Mathematical Institute of the Russian Academy of Sciences

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- Classical and non-classical Algebraic Structures,
- Algebra and Logics, Number Theory,
- Geometry and Topology, Analysis and Equations,
- Boolean and De Morgan functions, Cryptography and Discrete Mathematics, Applied Mathematics,
- Lattices, Universal Algebra, Computer Science and IT,
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- Quantum information theory, Quantum Logic and Quantum Computation, Quantum Groups and Quantum Quasigroups.

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CONTENTS

CHARACTERIZATION OF HYPERIDENTITIES DEFINED BY THE EQUALITIES $((x,y),u,v)=(x,(y,u),v)$ AND $((x,y),u,v)=(x,y,(u,v))$

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The following universal formula from a second-order language with specialized quantifiers have been studied in various domains of algebra and its applications and it was called hyperidentity:

$$\forall X_1,\ldots,X_m \forall x_1,\ldots,x_n (W_1 = W_2), \tag{1}$$

where w_1, w_2 are terms (words) in the functional variables $X_1,...,X_m$ and in the object variables $x_1,...,x_n$. For simplicity the hyperidentity is written without a quantifier prefix, i.e. as an equality: $w_1 = w_2$. The number m is called functional rank and the number n is called object rank of the given hyperidentity. A hyperidentity is true (or satisfied) in an algebra $(Q;U)$ if the equality $w_1 = w_2$ is valid when every object variable and every functional variable in it is replaced by any arbitrary element of Q and any operation of the corresponding arity from U respectively (it is assumed that such replacement is possible).

An algebra with binary and ternary operations is called {2,3}-algebra. A {2,3}-algebra $(Q;U)$ is called:

a) functionally non-trivial if the sets of its binary and ternary operations are non-singleton;

b) $2q$-algebra if there exists a binary quasigroup operation in U;

c) $3q$-algebra if there exists a ternary quasigroup operation in U;

d) invertible algebra if its every operation is a quasigroup operation.

In this talk we give the syntactic classification of functionally non-trivial hyperidentities which are defined by the equalities $((x,y),u,v)=(x,(y,u),v)$ or $((x,y),u,v)=(x,y,(u,v))$ and satisfied in above classes of algebras.

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LONG RANGE DIFFUSION-REACTION MODEL ON POPULATION DYNAMICS

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A model for long range diffusion-reaction on population dynamics has been created, then conditions for the existence and uniqueness of solutions of the model have been found in $L(p, q)$ norms.

CENTRAL EXTENSIONS OF FREE PERIODIC GROUPS

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We have proved that any countable abelian group $\mathcal{D}$ can be verbally embedded as a center in a m-generated group $A_{\mathcal{D}}$ such that the quotient group $A_{\mathcal{D}}/\mathcal{D}$ will be isomorphic to the free periodic group $B(m,n)$, where $m>1$ and $n\geq 665$ is an odd number. The proof is based on some generalization of the approach proposed by S. I. Adian in his monograph [1] for the positive solution of a long-standing open question in group theory: *is there a non-commutative group the intersection of any two non-trivial subgroups of which is infinite.*

For constructing the group $A_{\mathcal{D}}$ we fix an arbitrary countable abelian group

$$D=\langle d_1,d_2,...,d_i,...|r=1,r\in\mathcal{R}\rangle,$$

where $\mathcal{R}$ is some set of words in the group alphabet $d_1,d_2,...,d_i,....$

Consider the set of all elementary words $\mathcal{E}=\cup_{\alpha=1}^{\infty}\mathcal{E}_\alpha$ in the group alphabet $\{a_1,a_2,...,a_m\}$ defined in [1]. The set $\mathcal{E}$ is countable and let $\{A_j|j\in\mathbb{N}\}$ be some numeration of $\mathcal{E}$.

Denote by $A_{\mathcal{D}}(m,n)$ the group generated by

$$a_1,a_2,...,a_m,\ d_1,d_2,...,d_i,...,$$

and having the defining relations of the form

$$r=1 \text{ for all } r\in\mathcal{R},$$

$$a_id_j=d_ja_i \text{ for all } i=1,2,...,m\ j\in\mathbb{N},$$

$$A_j^n=d_j \text{ for all } A_j\in\mathcal{E}\ j\in\mathbb{N}.$$

The following theorem is true.

Theorem.

1. *In the group $A_{\mathcal{D}}(m,n)$ the identity $[x^n,y]=1$ holds,*
2. *verbal subgroups of the group $A_{\mathcal{D}}(m,n)$ corresponding to the word x^n coincide with the Abelian group $\mathcal{D}$,*
3. *the center of the group $A_{\mathcal{D}}(m,n)$ coincides with $\mathcal{D}$,*
4. *the quotient group of $A_{\mathcal{D}}(m,n)$ by the subgroup $\mathcal{D}$ is the free Burnside group $B(m,n)$.*

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ON VARIETIES OF TWO DIMENSIONAL REPRESENTATIONS OF A FAMILY OF ONE-RELATOR GROUPS

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Let $G = \langle g_1, \dots, g_m \rangle$ be a finitely generated group and $H \subset GL_n(K)$ a connected linear algebraic group defined over a field K which will be assumed to be algebraically closed and of characteristic zero. For any homomorphism $\rho : G \to H(K)$ the set of elements $(\rho(g_1), \dots, \rho(g_m)) \in H(K)^m$ satisfies evidently all the relations of G and thus the correspondence $\rho \mapsto (\rho(g_1), \dots, \rho(g_m))$ gives a bijection between points of the set $\hom(G, H(K))$ and K-points of some affine K-variety $R(G, H) \subset H^m$. The variety $R(G, H)$ is usually called the representation variety of G into the algebraic group H ([1]). In the case $G = GL_n(K)$ we will denote it simply by $R_n(G)$ and call it the variety of n-dimensional representations of G.

The study of geometric invariants of $R(G, H)$ like the dimension or the number of irreducible components is of interest in combinatorial group theory ([2]). The varieties of representations have also many applications in 3-dimensional geometry and topology ([3]).

Let us consider the group $G = \langle a, b \mid a^m = b^n \rangle$ where m and n are integers greater than one. We study the variety of representations $R_2(G)$. Let $d = \gcd(n, m)$. It is not difficult to see that a curve $x^n = y^m$ in $(K^*)^2$ has d irreducible components $U_1, \dots, U_d$. Let $\alpha, \beta \in K$ be elements such that $\alpha^n = \beta^m = 1$ and $\alpha \neq 1$, $\beta \neq 1$. Let us consider morphisms

$$f_{ij} : U_i \times U_j \times GL_2(K) \to R_2(G),$$
$$(x_1, y_1, x_2, y_2, A) \mapsto \left(A \begin{pmatrix} x_1 & 0 \\ 0 & x_2 \end{pmatrix} A^{-1}, A \begin{pmatrix} y_1 & 0 \\ 0 & y_2 \end{pmatrix} A^{-1} \right),$$

$$h_{i,\alpha,\beta} : U_i \times GL_2(K) \times GL_2(K) \to R_2(G),$$
$$(x, y, A, B) \mapsto \left(A \begin{pmatrix} x & 0 \\ 0 & \alpha x \end{pmatrix} A^{-1}, B \begin{pmatrix} y & 0 \\ 0 & \beta y \end{pmatrix} B^{-1} \right).$$

Let $V_{i,j}$ and $W_{i,\alpha,\beta}$ be closures of images of $f_{i,j}$ and $h_{i,\alpha,\beta}$ in Zarisski topology respectively. Then the following theorem holds.

Theorem. *1) Varieties $V_{i,j}$, where $1 \leq i, j \leq d$, and $W_{i,\alpha,\beta}$, where $1 \leq i \leq d$ and $\alpha^n = \beta^m = 1$ with $\alpha \neq 1$, $\beta \neq 1$, are all irreducible components of $R_2(G)$.*

2) The number of irreducible componets of the representation variety $R_2(G)$ is equal to $d^2 + d(n-1)(m-1)$.

3) $\dim V_{i,j} = 4$*,* $\dim W_{i,\alpha,\beta} = 5$ *for all* i, j, α, β*.*

4) All irreducible components of $R_2(G)$ are rational varieties.

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A SURVEY ON RESIDUAL TRANSCENDENTAL EXTENSIONS OF VALUATIONS

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Dedicated to the memory of Nicolae Popescu (1937-2010). In this paper we survey some results related to residual transcendental extensions of valuations.

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TWO RANDOM POINTS IN A CONVEX DOMAIN

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Complicated geometrical patterns occur in many areas of science. Their analysis requires creation of mathematical models and development of special mathematical tools. The corresponding area of mathematical research is called Stochastic Geometry. Among more popular applications are Stereology and Tomography (see [3]). The methods of form analysis are based on analysis of the objects as figures. For these sets, geometrical characteristics are considered that are independent of the position and orientation of the figures (hence they coincide for congruent figures). Classical examples are area and perimeter of a figure. In the last century German mathematician W. Blaschke formulated the problem of investigation of bounded convex domains in the plane using probabilistic methods. In particular, the problem of recognition of bounded convex domains $\mathbb{D}$ by chord length distribution. Let $\mathbb{G}$ be the space of all lines g in the Euclidean plane. Random lines generate chords of random length in convex domain $\mathbb{D}$. The corresponding distribution function is called the chord length distribution function

$$F_{\mathbb{D}}(x) = \frac{1}{|\partial\mathbb{D}|}\,\mu\{g \in \mathbb{G}:\ \chi(g) = g \cap \mathbb{D} \leq x\}$$

where $|\partial\mathbb{D}|$ is the perimeter of $\mathbb{D}$, and μ is invariant measure with respect to the group of Euclidean motions (translations and rotations). We choose uniformly and independently two points from $\mathbb{D}$. How large is the k-th moment of the Euclidean distance $\rho_k(\mathbb{D})$ between these two points? In other words, we need to calculate the quantity

$$\rho_k(\mathbb{D}) = \frac{1}{[S(\mathbb{D})]^2}\int\limits_{\mathbb{D}}\int\limits_{\mathbb{D}} \|Q_1 - Q_2\|^k \, dQ_1 \, dQ_2, \qquad k = 1, 2, 3, ...$$

where $S(\mathbb{D})$ is the area of $\mathbb{D}$, and $\|Q_1 - Q_2\|$ is the Euclidean distance between points Q_1 and Q_2. dQ_i, $i = 1, 2$ is an element of Lebesgue measure in the plane. The present problem was stated in [5] (see also [6]). We can rewrite $\rho_k(\mathbb{D})$ to the following form:

$$\rho_k(\mathbb{D}) = \frac{2\,|\partial\mathbb{D}|}{(k+2)(k+3)\,[S(\mathbb{D})]^2}\int\limits_0^{\infty} x^{k+3}\, f_{\mathbb{D}}(x)\, dx, \qquad k = 1, 2, 3...,$$

where $f_{\mathbb{D}}(y)$ is the density function of $F_{\mathbb{D}}(y)$. Therefore, if we know the explicit form of the length chord density function we can calculate the k-th moment of the distance between two random points in $\mathbb{D}$. It is not difficult to calculate $\rho_k(\mathbb{D})$ for a disc, regular triangle, a rectangle, a rhombus, a regular pentagon and regular hexagon. This formula allows to find an explicit form of k-th moment of the distance for those **D** for which the chord lenght distribution is known (see [1], [2] and [4]).

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DESCRIPTION OF CHAOTIC PATTERNS IN MODELS OF COUPLED OSCILLATORS IN TERMS OF MUTUAL INFORMATION

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Systems of coupled oscillators have become an object of intensive investigation recently. Synchronization within networks of oscillators is widespread in nature, though interpretation of links connecting the oscillators, their type and strenght are often subjective and depend on the model developer's vision of interaction mechanics. The hydrodynamics provides formidable models of formation of structures with increasing complexity, but ecology has also explored examples of such complexity, which include multiplicity of stable states and irregular dynamics. In fact ecological systems are never stable, at least in stability state by Lypunov. Over past decades many applications of such models in economics became widespread, while applications in various fields of science, such as chemistry, biology has much longer history. It is well-known that simple dynamical models can demonstrate complex behavior was established in the classical work of Lorents, later, in 1971 the concept of the strange attractor was introduced by Ruelle and Takens (e.g.[1]). The stochastic behavior of dynamical systems is called chaos, though it is important to underline that bifurcations and chaos stem from works of Poincar, who was the first person to discover a chaotic deterministic system which laid the foundations of modern chaos theory.

Investigators consider chaos as a model for studying transitive behavior in complex systems. The metric entropy serves an apparent criterion of complexity, since it specifies the average rate of a dynamical system orbit divergence. With respect to ecological models, particularly trophic chains, Yu. Svirezhev [1] brought an approximate formula for the entropy calculation and numerical investigation of the strange attractor for the chain of length three. An innumerable plenty of works exist that investigate approximate entropy as a measure of complexity and routes to chaos in different systems, we will not refer them here. In [2] it was demonstrated that the qualitative description of the multidimensional trophic chain to the system of coupled oscillators was given, which was used for extension of the persistence or ecostability region estimation and interpretation of a model phenomenon concerning the existence of so called paradoxical trophic chains. The work [3] was focused on the determination of transition times between the equilibriums due to random perturbations in multidimensional models. Other application of coupled oscillators dynamics to spatial ecological models are given in [4, 5,6].

The synchronization of chaotic oscillators, is a phenomenon that has been investigated intensively for the last two decades. As it is noted in [7]: "While the synchronization of chaotic oscillators with strange attractors has become familiar in the last two decades, most work on such systems has examined engineered systems, primarily for application to secure communications, using the low-dimensional signal connecting the oscillators as a carrier that is difficult to distinguish from noise".

This paper focuses on description of states in a system of the two oscillators with a unidirectional coupling. It is proposed in Fraser [8] that mutual information could provide a quantitative characterization of chaotic spatial patterns. The method includes considering messages as the values that measurements of attractors might take. It is an easy task to reinterpret the scheme of an ecological network to communication channel or a chain of connected channels, what are specific characteristics derived from such presentation has been and continue to be a matter of large discussion in literature. Let X and Y denote the oscillators 1 and 2 correspondingly. It is a simple observation that when the oscillators are sychronized the mutual information is equal to entropy, in conventional notations for the mutual information and entropy it is expressed in the form:

$$I(X, Y) = H(X) = H(Y).$$

In case they are not the inequality is:

$$I(X, Y) < \min[H(X),\ H(Y)].$$

In the first case the system produces trajectories that are indistingushable, to specify a strength of coupling a parameter taking values in the interval $[0, 1]$ is used. For the data on point oscillators we will consider a model where the data is available, in particular such as Roissler attractors. A purely statistical model of an ecological community was explored in [9].

We will implement the recursive method of calculating mutual information presented in [8] and apply it to identifying threshold parameter values for critical transitions in ecological networks.

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AN UPPER BOUND FOR THE COMPLEXITY OF COSET COVERING OF SUBSETS IN A FINITE FILELD

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Let F_q be a finite field with q elements, and F_q^n for an n-dimensional linear space over F_q (obviously F_q^n is isomorphic to F_{q^n}). If L is a linear subspace in F_q^n, then the set $\alpha + L \equiv \{\alpha + x | x \in L\}$, $\alpha \in F_q^n$ is a coset (or translate) of the subspace L and $\dim(\alpha + L)$ coincides with $\dim L$. An equivalent definition: a subset $N \subseteq F_q^n$ is a coset if whenever $x^1, x^2, \dots, x^m$ are in N, so is any affine combination of them, i.e., so is $\sum\limits_{i=1}^{m} \lambda_i x^i$ for any $\lambda_1, \dots, \lambda_m$ in F_q such that $\sum\limits_{i=1}^{m} \lambda_i = 1$. It can be readily verified that any k-dimensional coset in F_q^n can be represented as a set of solutions of a certain system of linear equations over F_q of rank $n - k$ and vice versa.

Definition 1. *A set* M *of cosets C form a coset covering for a subset N in F_q^n iff $N = \bigcup\limits_{C \in \mathrm{M}} C$. The number of cosets in* M *is the length (or complexity) of the covering. The shortest coset covering is the covering of the minimal possible length.*

The problem of finding of the shortest coset covering originally was considered in F_2^n in relation with a natural generalization of the notion of Disjunctive Normal Forms of Boolean functions. A subset $N \subseteq F_q^n$ can be given in different ways: as a list of elements, as a set of solutions of a polynomial equation over F_q^n etc. Finding the shortest coset covering means finding the minimal number of cosets of linear subspaces (i.e. systems of linear over F_q equations), such that N coincides with the union of those cosets.

We establish an upper bound for the length of the shortest coset covering based on some properties of the stabilizer of the subset N, considering the action of the General Affine Group on F_q^n.

Consider affine transformations of F_q^n of the form $y = xA + b$, where x, y and $b \in F_q^n$, and A is an $(n \times n)$-dimensional non-degenerate matrix over F_q. We refer to an affine transformation as a pair (A, b).The General Affine Group act naturally on F_q^n, on the set of all subsets in F_q^n and on the set of all cosets in F_q^n and coset dimension remain invariant under this action. Thus, if two subsets N_1 and N_2 are in the same orbit then, obviously, any coset covering for N_1 can be transformed to a coset covering of the same length for N_2 by an appropriate affine transformation, and coset covering properties are invariant under the action of the General Affine Group.

Definition 2. *A set* T. *of affine transformations is a coset if whenever* (A_1, b_1), (A_2, b_2), ...,(A_m, b_m) *are in* T, *so is* $\left(\sum_{i=1}^{m} \lambda_i A_i, \sum_{i=1}^{m} \lambda_i b_i\right)$ *for any* $\lambda_1, \ldots, \lambda_m$ *in* F_q *such that* $\sum_{i=1}^{m} \lambda_i = 1$.

For a given set of affine transformations one can consider coset covering and the shortest coset covering.

Definition 3. *Let G be a subgroup in the General Affine Group. The coset rank of G is the length of its shortest coset covering, which is denoted by $CR(G)$.*

Let $N \subseteq F_q^n$ and $Stab(N)$ be the stabilizer of N under the action of the General Affine Group. Any subgroup G in the stabilizer $Stab(N)$ act on N splitting N into disjoint orbits of elements. We denote the number of orbits by $\#orb_G(N)$.

Theorem. *The length of the shortest coset covering for a set $N \subseteq F_q^n$ is not greater than $CR(G) \times \#orb_G(N)$ for any subgroup G in $Stab(N)$. This upper bound is achievable and cannot be improved.*

THE SINE REPRESENTATION OF A CONVEX BODY

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The problem of the sin representation for the support function of a centrally symmetric convex body is studied. The article defines a subclass of centrally symmetric convex bodies which is dense in the class of centrally symmetric convex bodies. Also an inversion formula for the sin transform is found.

PROPERTIES OF SURJECTIVE REAL QUADRATIC MAPS

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The properties of surjective real quadratic maps are investigated. Sufficient conditions for the property of surjectivity to be stable under various perturbations are obtained. Examples of surjective quadratic maps whose surjectivity breaks down after an arbitrarily small perturbation are constructed. Sufficient conditions for quadratic maps to have nontrivial zeros are obtained. For a smooth even map in a neighborhood of the origin an inverse function theorem in terms of the degree of the corresponding quadratic map is obtained.

REDUCED C^*-ALGEBRA OF A GROUP GRADED SYSTEM

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The concept of group grading arises naturally in considering the crossed products, especially in the context of irreversible dynamical systems.

In the talk some general aspects concerning group graded systems are considered. The starting point were the paper [1] and the remarkable book of Exel [2] devoted to Fell C*-bundles.

We introduce the notion in an equivalent way based on a semigroup with a special structure. Naimely, if Γ is a discrete Abelian group then an involutive semigroup $\mathfrak{A}$ is called Γ-graded system, if it is a union of Banach spaces $\mathfrak{A}_\gamma,\ \gamma \in \Gamma$, intersecting only at 0, the operations of multiplication and involution on the semigroup being consistent with the linear operations on the component Banach spaces, and

(i) $ab \in \mathfrak{A}_{\alpha\beta}$ for $a \in \mathfrak{A}_\alpha,\ b \in \mathfrak{A}_\beta$, (iii) $\|ab\| \le \|a\|\|b\|$ for $a \in \mathfrak{A}_\alpha,\ b \in \mathfrak{A}_\beta$,

(ii) $a^* \in \mathfrak{A}_{\gamma^{-1}}$ for $a \in \mathfrak{A}_\gamma$, (iv) $\|a^*a\| = \|a\|^2 = \|a^*\|^2$ for $a \in \mathfrak{A}_\gamma$.

Obviously, the *central* algebra $A = \mathfrak{A}_e$ (e being a neutral element of Γ) is a C*-algebra as well as an involutive subsemigroup of the semigroup $\mathfrak{A}$.

The notions of graded subsystem, ideal, moduls, and morphisms between the graded systems are introduced in a natural way. Moreover, there is a standard Hilbert module structure on a Γ-graded system, an inner product defining as

$$< \xi,\ \eta > = \sum_{\gamma\in\Gamma} \eta_\gamma^* \xi_\gamma,$$

for $\xi,\ \eta \in \mathfrak{A},\ \xi = \{\xi_\gamma\},\ \eta = \{\eta_\gamma\}$.

There is a standard (regular) representation of the graded system in an associated Hilbert module, which we call the *reduced* C*-algebra.

We present a functional description of this algebra, realizing it as an algebra of continuous $\mathfrak{A}$-mappings on the dual group of Γ.

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ON ENDOMORPHISMS OF CC GROUPS

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We have obtained the description of the automorphisms of semigroups $\mathrm{End}\, G$ of groups G having only cyclic centralizers of nontrivial elements. The question of describing the automorphisms of $\mathrm{End}(A)$ for a free algebra A in a certain variety was considered by different authors since 2002 (see, for example, [1]–[4]). In particular, we prove that each automorphism of the automorphism group $\mathrm{Aut}(G)$ of groups G from this class is uniquely determined by its action on the elements from the subgroup of inner automorphisms $\mathrm{Inn}(G)$. The obtained general result includes the following cases: absolutely free groups, free Burnside groups of odd period $n \geq 665$, free groups of some infinitely based varieties (the cardinality of the set of such varieties is continuum), and so on.

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NEW INVESTIGATING METHOD IN THE PROBLEMS HALL AND SUM OF THREE CUBES

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Two well-known problems are considered in the report, one of them is the presentation of integer by the sum of three cubes, i.e. the solution of the Diophantine Equation $a^3 + b^3 + c^3 = d$, and the other is the Hall problem and its connection with the Davenport-Zanier polynomials. In this talk,we consider the problem in more general formulation: to find polynomials $P1(y)$, $P2(y)$, $P3(y)$ with the highest possible degree and $Q(y)$ with the lowest possible degree, such that the equality $P1(y)^3 + P2(y)^3 + P3(y)^3 = Q(y)(P1(y)^2 - P2(y)^3 = Q(y))$ holds. These issues are closely linked to each other. Using this method, computer solutions have been built for some specific cases of that problems. Nevertheless, there are more interesting cases that addressed to the solution of problems related to elliptic curves having applications in coding.

Keywords: Diophantine equations; sum of three cubes; Hall problem; parametric solutions; elliptic curves; Davenport-Zanier polynomials.

ON SOME TRENDS AND PRINCIPLES RELATED TO ARBITRARY MEROMORPHIC OR ANALYTIC FUNCTIONS IN A GIVEN DOMAIN

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The first results (priciples) related to arbitrary meromorphic, particularly analytic, functions in a given domain were established by Cauchy (19-th century), while the next results arisen a century later in Ahlfors theory of covering surfaces (1935).

In this survey we present some other (diverse type) results of the same generality which were obtained since 1970s.

The majority of these results occur in three trends in theory of meromorphic functions: Gamma-lines, proximity property, and universal version of value distribution theory.

Each of these trends complements the classical Nevanlinna value distribution theory or Ahlfors theory and also reveals some new type of phenomena.

Content: list of sections.

(The results in each section relate to arbitrary meromorphic or analytic functions in a given domain.)

1. Two principles related to derivatives.
2. Results related to level sets and Gamma-lines.
3. Three simple consequences related to a-points.
4. Ahlfors fundamental theorems in terms of windings and a new interpretation of deficient values.
5. Universal version of value distribution.

This work was supported by Marie Curie (IIF) award

THE TYPE PROBLEM AND THE GEOMETRY OF RIEMANN SURFACES

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While the geometric theory of finite type Riemann surfaces is well developed, the geometric study of infinite type (that is, infinitely generated fundamental group) Riemann surfaces is still in its infancy. In this talk we first describe some of the known results on the geometry and topology of infinite type surfaces and then discuss new results involving the relationship between the hyperbolic geometry of the Riemann surface and a version of the classical type problem (whether or not the surface carries a Green's function). Our tools include finding the relationship between the extremal length of curve families leaving the Riemann surface and the growth rates of closed geodesics on the surface.

GROUPS OF HOMEOMORPHISMS OF THE LINE AND THE CIRCLE. CRITERIA FOR ALMOST NILPOTENCY

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In the report, for finitely generated groups of homeomorphisms of the line and the circle in terms of free subsemigroups with two generators and the maximality condition, a criterion for almost nilpotency is obtained. Earlier, the criteria for almost nilpotency were also established for the finitely generated groups of diffeomorphisms of the line and the circle of C^1 smoothness with mutually transversal elements in terms of free subsemigroups with two generators. Moreover, in the case of groups of diffeomorphisms it was possible to obtain structural theorems and to show the typical character of a number of characteristics of such groups [1,2]. It is established that in the space of all finitely generated groups of diffeomorphisms with a given number of generators and of C^1 smoothness, the set of groups with mutually transversal elements contains a countable intersection of open everywhere dense subsets (a massive set) [3,4]. In the paper [5], for a finitely generated group of diffeomorphisms of the C^{1+a}, $a > 0$ smoothness interval in terms of free subsemigroups with two generators, Navas also established a criterion for almost nilpotency.

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MATRIX RINGS AS ONE SIDED $\sigma - (S, 1)$ RINGS

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Let R be a ring and σ an endomorphism of R. We recall that R is called an $(S, 1)$-ring if for $a, b \in R$, $ab = 0$ implies $aRb = 0$. We involve σ to generalize this notion. We say that R is a left $\sigma - (S, 1)$ ring if for $a, b \in R$, $ab = 0$ implies aRb =0 and $\sigma(a)Rb = 0$. We say that R is a right $\sigma - (S, 1)$ ring if for $a, b \in R$, $ab = 0$ implies $aRb = 0$ and $aR\sigma(b) = 0$. R is called a $\sigma - (S, 1)$ ring if it is both right and left $\sigma - (S, 1)$ ring. In this paper we give examples of such rings and a relation between $\sigma - (S, 1)$ rings and 2-primal rings.

We show that a certain class of matrix rings, with suitable endomorphisms σ are left $\sigma - (S, 1)$ but not right $\sigma - (S, 1)$, and vice versa.

2010 Mathematic Subject Classification: 16S 36, 16N 40, 16P 40, 16W 20.

Keywords and phrases: 2-primal, associated prime, automorphism, left $\sigma - (S, 1)$ ring, Ore extensions.

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ON OPTIMAL CRYPTOGRAPHIC FUNCTIONS

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We will give a brief overview of the recent progress on optimal cryptographic functions such as almost perfect nonlinear (APN) and almost bent (AB) functions. When used as S-boxes in block ciphers these vectorial Boolean functions possess the best possible resistance against the two main crypto attacks differential and linear attacks. However, the interest to these functions is not restricted to cryptography because they define optimal objects in different areas of mathematics and information theory such as coding theory, sequence design, commutative algebra and finite geometry.

Keywords: Boolean function, almost bent, almost perfect nonlinear (APN), equivalence of functions,

POSITIVE RESOLUTION OF KRAUTER CONJECTURE ON PERMANENTS

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The class of $(-1, 1)$-matrices is very important in algebra and combinatorics and in various their applications. For example, well-known Hadamard matrices are of this type.

An important matrix function is the permanent:

$$\operatorname{per} A = \sum_{\sigma \in S_n} a_{1\sigma(1)} \cdots a_{n\sigma(n)},$$

here $A = (a_{ij}) \in M_n(\mathbb{F})$ is an $n \times n$ matrix over a field $\mathbb{F}$ and S_n denotes the set of all permutations of the set $\{1, \dots, n\}$.

While the computation of the determinant can be done in a polynomial time, it is still an open question, if there are such algorithms to compute the permanent.

In this talk we discuss the permanents of ± 1-matrices.

In 1974 Wang [2, Problem 2] posed a problem to find a decent upper bound for $|\operatorname{per}(A)|$ if A is a square ± 1-matrix of rank k. In 1985 Kräuter [1] conjectured some concrete upper bound.

We prove the Kräuter's conjecture and thus obtain the complete answer to the Wang's question. In particular, we characterized matrices with the maximal possible permanent for each value of k.

The work is partially financially supported by RFBR grant 17-01-00895.

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CONDITIONAL MEASURES OF DETERMINANTAL POINT PROCESSES: THE GIBBS PROPERTY AND THE COMPLETENESS OF REPRODUCING KERNELS

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Consider a Gaussian Analytic Function on the disk. In joint work with Yanqi Qiu and Alexander Shamov, we show that, almost surely, there does not a square-integrable holomorphic function with the same zeros. By the Peres and Virag Theorem, zeros of a Gaussian Analytic Function on the disk are a determinantal point process governed by the Bergman kernel, and we prove, for general determinantal point processes, that reproducing kernels sampled along a trajectory form a complete system in the ambient Hilbert space. The key step in our proof is that the determinantal property is preserved under conditioning.

The talk will first address this question for specific examples such as the sine-process, where one can explicitly write the analogue of the Gibbs condition in our situation. We will then consider the general case, where, in joint work with Yanqi Qiu and Alexander Shamov, proof is given of the Lyons-Peres conjecture on completeness of random kernels.

The talk is based on the preprint arXiv:1605.01400 as well as on the preprint arXiv:1612.06751 joint with Yanqi Qiu and Alexander Shamov.

ON NEW BASIS PROPERTIES OF REGULAR AND REAL FUNCTIONS: PARITY, ANTIHOLOMORPHY, ABSTRACT AFINITE **C**-TREE AND POLYHOLOMORPHY

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For every holomorphic real function $f(z)$ (real on a real subset), the real $u(x,y)$ and imaginary $v(x,y)$ parts are respectively even and odd in y. In the proof uses the Cauchy-Riemann conditions. Even or odd (thinned) power series are derived, especially compact ones for one and two (y_1, y_2) arguments. The even and odd functions $f(z)$ or their even-odd components adequately correspond to their real and imaginary parts (even-odd both in y and x; one or many $z_{1,\ldots,m}$). The simple elegant asymptotes for small y and for some critical points (previously applied by the author in the theory of waves) are presented. The conjugation of the argument $\bar{z}$ goes into the antiholomorphy of $\bar{f}(z)$. The real and imaginary parts of any regular function $\varphi(z)$ are actually and trivial real and holomorphic in two variables x and y, again complex, (or doubled, for $m \geqslant 2$). And their real and imaginary parts in turn by 4 or $4m$ arguments. And so on, unboundedly, bi-, 4-, 8-, ..., poly-holomorphy on sets of the complex hyperspace $\mathbf{C}^p$ ($2p$ axes, $p = m2^{\kappa-1}$, $\kappa = 1, 2, \ldots$) or simply on the **C**-tree: $z = x+iy$, $x = x_{X2}+iy_{X2}$, $y = x_{Y2}+iy_{Y2}$, $x_{X2} = x_{X3}+iy_{X3}, \ldots$ (Here $\mathbf{C}^p$ in some contrast from n-dimensional $\mathbf{C}^n$, polyholomorphicity from pluriregularity). And all these parts and functions possess the formulated fundamental properties of regular real functions. The established polyholomorphy and the **C**-tree are purely abstract, have not receive applications.

ALGEBRAIC IDEAS IN DYNAMICAL SYSTEMS

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Symbolic dynamics is a fascinating branch of ergodic theory and dynamical systems, and cellular automata constitute its central topic. In this talk, I'll survey some results obtained in collaboration with Michel Coornaert focusing on the algebraic aspects of cellular automata and of some other, closely related, dynamical systems.

WORD AND CONJUGACY PROBLEMS IN FINITELY GENERATED GROUPS

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In early 1970's Donald Collins posed a well-known question about possibility of embedding torsion-free groups with decidable word problem in groups with decidable conjugacy problem. We answer this question by showing that there exist torsion-free finitely presented and finitely generated solvable groups with decidable word problem which do not embed in groups with decidable conjugacy problem.

Generalizing our approach in different directions we are able to obtain other interesting results as well as answer other open questions.

SOME REMARKS ON MANOUSSAKIS' CONJECTURE FOR A DIGRAPH TO BE HAMILTONIAN

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Terminology and notations below follows [1]. We consider finite digraphs without loops and multiple arcs. A digraph D is hamiltonian (traceable) if it contains a cycle (a path) through all its vertices. Manoussakis [4] proposed the following conjecture.

Conjecture 1 ([4]). *Let G be a strongly 2-connected digraph of order n such that for all distinct pairs of non-adjacent vertices x, y and w, z we have $d(x) + d(y) + d(w) + d(z) \geq 4n - 3$. Then D is hamiltonian.*

By the theorems of Fraisse and Thomassen [3], and Meyniel [5] the conjecture is true when D contains at most one pair of non-adjacent vertices or $d(x)+d(y) \geq 2n-1$ for all pairs of non-adjacent vertices x, y, respectively. From a result by Darbinyan [2] it follows that if a digraph D satisfies the conditions of Conjecture 1, then it contains a cycle of length at least $n-1$, in particular, D is traceable.

Let D be a digraph satisfying the conditions of Conjecture 1. Moreover, assume that D contains a pair of non-adjacent vertices x_0, y_0 such that $d(x_0)+d(y_0) \leq 2n-k$, where $k \geq 2$. Notice that for every pair of non-adjacent vertices $\{x, y\}$ other than $\{x_0, y_0\}$, $d(x) + d(y) \geq 2n + k - 3$.

In this paper we prove the following theorems.

Theorem 1. *The conjecture is true if and only if D contains a cycle through x_0 and y_0.*

Theorem 2. *If $d(x_0) \geq n - 4$ or $d(y_0) \geq n - 4$, then the conjecture is true, i.e., D is hamiltonian. (In particular, for all n, $n \leq 15$, the conjecture is true).*

Theorem 3. *The digraph D is hamiltonian or contains cycles of all lengths m, $2 \leq m \leq n - 1$.*

Theorem 4. *The digraph D is hamiltonian or for any $z_0 \in \{x_0, y_0\}$ there is a cycle of length $n - 1$, which does not contain z_0.*

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MAGIC ACTION OF o-POLYNOMIALS AND EA-EQUIVALENCE OF NIHO BENT FUNCTIONS

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Boolean functions of n variables are binary functions over the vector space F_2^n of all binary vectors of length n. Bent functions, introduced by Rothaus [1] in 1976, are Boolean functions of even number of variables n, that are maximally nonlinear in the sense that their nonlinearity, the minimum Hamming distance to all linear functions, is optimal. Bent functions have attracted a lot of research interest in mathematics because of their relation to difference sets and to designs, and in the applications of mathematics to computer science because of their relations to coding theory and cryptography. In general, bent functions are considered up to EA-equivalence, that is, functions within one class can be obtained from each other by composition from the left side by an affine permutation and by adding an affine Boolean function.

It is proven in [2] that so-called, Niho bent functions, introduced in [3], define $o-$polynomials and, conversely, every $o-$polynomial defines a Niho bent function. As further observed in the same paper, the projective equivalence of $o-$polynomials defines, for Niho bent functions, an equivalence relation called o-equivalence and, in general, the two o-equivalent Niho bent functions defined from an o-polynomial F and its inverse F^{-1} are $EA-$inequivalent. The study of $o-$equivalence was further continued in [4]. In that paper a group of transformations of order 24 preserving projective equivalence and introduced in [5] was in focus and it was discovered that there are two more transformations preserving $o-$equivalence but providing $EA-$inequivalent bent functions.

In our work we study so-called magic action, a transformation of $o-$polynomials preserving projective equivalence introduced in [6]. We prove that this transformation does not provide further new $EA-$inequivalent bent functions.

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PARASTROPHICALLY UNCANCELLABLE EQUATIONS WITH DIVISION AND REGULAR OPERATIONS

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We consider 48 parastrophically uncancellable quadratic functional equations with four object variables and two division and regular operations in two classes: balanced non–Belousov (consists of 16 equations) and non–balanced non–gemini (consists of 32 equations). An endo-linear representation on the group (Abelian group) for a pair of division and regular operations satisfying one of these parastrophically uncancellable quadratic equations is obtained. As a consequence of these results, an endo-linear representation for every operation of a binary algebra satisfying one of these hyperidentities is given.

INVERTIBLE BINARY ALGEBRAS ISOTOPIC TO A GROUP OR AN ABELIAN GROUP

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A binary algebra $(Q;\Sigma)$ is called invertible algebra or system of quasigroups if each operation in Σ is a quasigroup operation. Invertible algebras with second order formulas first were considered by Shaufler in connection with coding theory. He pointed out that the resulting message would be more difficult to decode by unauthorized receiver than in the case when a single operation is used for calculation.

We obtained characterizations of invertible algebras isotopic to a group or an abelian group by the second-order formula.

Definition. We say that a binary algebra $(Q;\Sigma)$ is isotopic to the groupoid $Q(\cdot)$, if each operation in Σ is isotopic to the groupoid $Q(\cdot)$, i.e. for every operation $A\in\Sigma$ there exists permutations α_A, β_A, γ_A of Q, that:

$$\gamma_A A(x,y)=\alpha_A x\cdot\beta_A y,$$

for every $x,y\in Q$. Isopoty is called principal if $\gamma_A=\epsilon$ (ϵ - unit permutation) for every $A\in\Sigma$.

Theorem 1. *The invertible algebra $(Q;\Sigma)$ is a principally isotopic to a group, if and only if the following second-order formula*

$$A({}^{-1}A(B(x,B^{-1}(y,z)),u),v)=B(x,B^{-1}(y,A({}^{-1}A(z,u),v))),$$

is valid in the algebra $(Q;\Sigma\cup\Sigma^{-1}\cup{}^{-1}\Sigma)$ for all $A,B\in\Sigma$.

Corollary 1. *The class of quasigroups isotopic to groups is characterized by the following identity:*

$$x(y\backslash((z/u)v))=((x(y\backslash z))/u)v.$$

Theorem 2. *The invertible algebra $(Q;\Sigma)$ is a principally isotopic to an abelian group if and only if the following second-order formula:*

$$A({}^{-1}A(B(x,z),y),A^{-1}(u,B(w,y)))=$$
$$=A({}^{-1}A(B(w,z),y),A^{-1}(u,B(x,y))).$$

is valid in the algebra ($(Q;\Sigma\cup\Sigma^{-1}\cup{}^{-1}\Sigma)$ for all $A,B\in\Sigma$.

Corollary 2. *The class of quasigroups isotopic to abelian groups is characterized by the following identity:*

$$((xz)/y)(u\backslash(wy))=((wz)/y)(u\backslash(xy)).$$

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USING OF 2D HAAR WAVELETS FOR SOLVING OF MIXED 2D NONLINEAR FREDHOLM VOLTERRA INTEGRAL EQUATION

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In this paper, we suggest introduce a new and efficient numerical approach for solving of mixed 2D nonlinear Fredholm - Volterra integral equations. The fundamental structure of this method is based on the using of 2D Haar wavelet. Also, error analysis for method is presented by using the Banach fixed point theorem, and this theorem guarantees that under certain assumptions, this equation has a unique fixed point. Finally, some numerical examples are given to show the accuracy of the method, and results are compared with other numerical methods.

2010 MSC: 45P99, 65T60, 37C25.

Keywords: Nonlinear integral equation; Rationalized Haar wavelet; fixed point theorem.

AUTOMORPHISM GROUPS OF SUPEREXTENSIONS OF SEMIGROUPS

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The through study of various extensions of semigroups was started in [12] and continued in [1]-[10], [13]-[19]. The largest among these extensions is the semigroup $\upsilon(S)$ of all upfamilies on a semigroup S. A family $\mathcal{M}$ of non-empty subsets of a set X is called *an upfamily* if for each set $A \in \mathcal{M}$ any subset $B \supset A$ of X belongs to $\mathcal{M}$. Each family $\mathcal{B}$ of non-empty subsets of X generates the upfamily $\{A \subset X : \exists B \in \mathcal{B}\ (B \subset A)\}$ which we denote by $\langle B \subset X : B \in \mathcal{B}\rangle$. An upfamily $\mathcal{F}$ that is closed under taking finite intersections is called a *filter*. A filter $\mathcal{U}$ is called an *ultrafilter* if $\mathcal{U} = \mathcal{F}$ for any filter $\mathcal{F}$ containing $\mathcal{U}$. The family $\beta(X)$ of all ultrafilters on a set X is called the *Stone-Čech compactification* of X, see [20]. An ultrafilter $\langle\{x\}\rangle$, generated by a singleton $\{x\}$, $x \in X$, is called *principal*. Each point $x \in X$ is identified with the principal ultrafilter $\langle\{x\}\rangle$ generated by the singleton $\{x\}$, and hence we can consider $X \subset \beta(X) \subset \upsilon(X)$. It was shown in [12] that any associative binary operation $* : S \times S \to S$ can be extended to an associative binary operation $* : \upsilon(S) \times \upsilon(S) \to \upsilon(S)$ by the formula

$$\mathcal{L} * \mathcal{M} = \Big\langle \bigcup_{a \in L} a * M_a : L \in \mathcal{L},\ \ \{M_a\}_{a\in L} \subset \mathcal{M} \Big\rangle$$

for upfamilies $\mathcal{L}, \mathcal{M} \in \upsilon(S)$. In this case the Stone-Čech compactification $\beta(S)$ is a subsemigroup of the semigroup $\upsilon(S)$. The semigroup $\upsilon(S)$ contains as subsemigroups many other important extensions of S. In particular, it contains the semigroup $\lambda(S)$ of maximal linked upfamilies, see [11, 12]. An upfamily $\mathcal{L}$ of subsets of S is said to be *linked* if $A \cap B \neq \emptyset$ for all $A, B \in \mathcal{L}$. A linked upfamily $\mathcal{M}$ of subsets of S is *maximal linked* if $\mathcal{M}$ coincides with each linked upfamily $\mathcal{L}$ on S that contains $\mathcal{M}$. It follows that $\beta(S)$ is a subsemigroup of $\lambda(S)$. The space $\lambda(S)$ is well-known in General and Categorial Topology as the *superextension* of S, see [21, 22].

Given a semigroup S we shall discuss the algebraic structure of the automorphism group $\mathrm{Aut}\,(\lambda(S))$ of the superextension $\lambda(S)$ of S. We show that any automorphism of a semigroup S can be extended to an automorphism of its superextension $\lambda(S)$, and the automorphism group $\mathrm{Aut}\,(\lambda(S))$ of the superextension $\lambda(S)$ of a semigroup S contains a subgroup, isomorphic to the group $\mathrm{Aut}\,(S)$. We describe automorphism groups of superextensions of groups, finite monogenic semigroups, null semigroups, almost null semigroups, right zero semigroups, left zero semigroups and all three-element semigroups.

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ON MEDIAL STRUCTURES

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In this talk we characterize the structure of invertible algebras, $(Q;\Sigma)$, with the following $\forall\exists(\forall)$-identities of mediality from the second order Logic:

$$\forall X,Y\exists X',Y',Z'\forall x,y,u,v\left(X(Y(x,y),X(u,v))=X'(Y'(x,u),Z'(y,v))\right), \tag{1}$$

$$\forall X,Y\exists X',Y',Z'\forall x,y,u,v\left(X(X(x,y),Y(u,v))=X'(Y'(x,u),Z'(y,v))\right), \tag{2}$$

$$\forall X,Y\exists X',Y',Z'\forall x,y,u,v\left(X(Y(x,y),Y(u,v))=X'(Y'(x,u),Z'(y,v))\right). \tag{3}$$

Let Ω_Q be the set of all binary quasigroup operations on the set Q.

We say that algebra $(Q;\Sigma)$ satisfies the $\forall\exists^*(\forall)$-identity of mediality (1) if for every pair of operations $A,B\in\Sigma$ there exists a triple of operations $A',B',C'\in\Omega_Q$ such that

$$A(B(x,y),A(u,v))=A'(B'(x,u),C'(y,v))$$

for every $x,y,u,v\in Q$.

Theorem 1. *If invertible algebra $(Q;\Sigma)$ satisfies the $\forall\exists^*(\forall)$-identity of mediality (1) then there exists an abelian group $Q(\circ)$ such that any operation $A_i\in\Sigma$ is determined by the rule:*

$$A_i(x,y)=\varphi_i x\circ t_i\circ\psi_i y,$$

where $\varphi_i,\psi_i\in\operatorname{Aut}Q(\circ)$ and $t_i\in Q$.

The similar results are valid for the other considered second order formulas.

Corollary 1. *If in algebra $Q(\cdot,A,B,C)$ with four quasigroup operations is satisfied the identity:*

$$(x\cdot y)\cdot(u\cdot v)=A(B(x,u),C(y,v)),$$

then there exists an abelian group $Q(+)$ such that

$$x\cdot y=\varphi x+t+\psi y,$$

where $\varphi,\psi\in\operatorname{Aut}Q(+)$ and $t\in Q$.

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DIMENSION OF SHAPE MAPS

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If $(f_\mu, \phi) : \mathbf{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda) \to \mathbf{Y} = (Y_\mu, q_{\mu\mu'}, M)$ is a morphism of inverse systems in the homotopy category of polyhedra HPol, and if $(\lambda, \mu) \in \Lambda \times M$, with $\lambda \geq \phi(\mu)$, denote $f_{\mu\lambda} := f_\mu \circ p_{\phi(\mu)\lambda}$.

Definition 1. We say that a morphism of inverse systems $(f_\mu, \phi) : \mathbf{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda) \to \mathbf{Y} = (Y_\mu, q_{\mu\mu'}, M)$ in the category HPol has dimension $\dim(f_\mu, \phi) \leq n$ if every $\mu \in M$ admits a $\lambda \geq \phi(\mu)$ such that the H-map $f_{\mu\lambda} : X_\lambda \to Y_\mu$ factors in HPol through a polyhedron P with $\dim P \leq n$, i.e., there are H-maps $u : X_\lambda \to P$, $v : P \to Y_\mu$ such that $f_{\mu\lambda} = v \circ u$.

Definition 2. A shape morphism $F : X \to Y$ between topological spaces has *shape dimension* $\operatorname{sd} F \leq n$, $n \geq 0$, provided F admits a representation $(f_\mu, \phi) : \mathbf{X} \to \mathbf{Y}$ with $\dim(f_\mu, \phi) \leq n$.

Particularly, a map $f : X \to Y$ has shape dimension $\operatorname{sd} f \leq n$ if its shape image by the shape functor $S : HTop \to Sh$ has $sd(S(f)) \leq n$.

We put $\operatorname{sd} F = n$ (or $\operatorname{sd} f = n$) provided n is the least m for which $\operatorname{sd} F \leq m$ (resp. $\operatorname{sd} f \leq m$).

Theorem 1. *A topological space X has shape dimension $\operatorname{sd} X \leq n$ if and only if the identity map of X has shape dimension $\operatorname{sd}(1_X) \leq n$.*

Theorem 2. *Let $F : X \to Y$ be a shape morphism of topological spaces. If $\operatorname{sd} X \leq n$ or $\operatorname{sd} Y \leq n$ then $\operatorname{sd} F \leq n$.*

Corollary 1. *Let $f : X \to Y$ be a continuous map. If $\dim X \leq n$ or $\dim Y \leq n$, then $\operatorname{sd} f \leq n$.*

The following theorem characterizes the shape dimension of map.

Theorem 3. *A map $f : X \to Y$ has shape dimension $\operatorname{sd} f \leq n$ if and only if for every map $h : Y \to P$ into a space $P \in HPol$, the composition $h \circ f$ homotopically factors through a polyhedron P' with $\dim P' \leq n$, i.e., there are maps $u : X \to P'$ and $v : P' \to P$ such that $h \circ f \simeq v \circ u$.*

Theorem 4. *Let $\mathbf{f} : \mathbf{X} \to \mathbf{Y}$ be a pro-morphism in the category HPol. If $\dim \mathbf{f} \leq n$, then for every Abelian group G and an index $k > n$ the homology pro-morphism $H_k(\mathbf{f}; G) : H_k(\mathbf{X}; G) \to H_k(\mathbf{Y}; G)$ is a zero-morphism of pro-groups.*

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EXISTENCE OF MAXIMUM ENTROPY PROBLEM SOLUTION IN A GENERAL N-DIMENSIONAL CASE

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Maximum entropy methodology applied in European call options seeks a risk neutral probability measure p, such that

$$Ap = b \tag{1}$$

$$\sum_{i=1}^{n} p_i = 1, \quad p_i \geq 0 \tag{2}$$

$$S(p) = -\sum_{i=1}^{n} p_i ln(p_i) \quad \text{is maximal} \tag{3}$$

where b is the vector of current option prices' future values for each strike and A is the matrix of pay-offs. We denote A's columns by $a_0, a_1, \ldots, a_n$ (note that $a_n = a_{n-1} + tI$ for some t). Consider the following $n+1$ hyperplane - vector pairs (we denote hyperplanes by $hp(\cdot)$ and convex hulls by $conv(\cdot)$).

$$\begin{cases} hp(a_1, a_2, \ldots, a_{n-1}, I), \quad a_0 \\ \vdots \\ hp(a_0, a_1, \ldots, a_{n-2}, a_{n-1}), \quad I \end{cases} \tag{4}$$

For each hyperplane we denote by N_i its normal "pointing" in the direction of the associated vector a_i. It is obvious that there exists a finite t, s.t. 1, 2 are satisfied if and only if the following inequalities take place.

$$\begin{cases} \langle N_0 - a_1, b - a_1 \rangle \geq 0 \\ \vdots \\ \langle N_n, b \rangle \geq 0 \end{cases} \tag{5}$$

Assuming that condition 5 is true, the following lemmas and the corresponding theorem are true.

Lemma 1. *$\exists \mu > 0$, s.t. $\forall t$ for which $b \in conv(a_0, \ldots, a_{n-1}, a_n)$, $t \geq \mu > 0$.*

Lemma 2. *If for some t_0 $b \in conv(a_0, \ldots, a_n)$, then this also holds for any $t > t_0$.*

Lemma 3. *Let T be the set of all t's, s.t. $b \in conv(a_0, \ldots, a_n)$, then* $\underline{t} = \inf T \in T$.

Theorem 1. *If condition 5 is satisfied, the angle between b and I isn't 0 and $b_n > 0$, then $b \in conv(a_0, \ldots, a_n)$, where $a_n = a_{n-1} + \underline{t}I$ and the factor of a_{n-1} is 0 in the linear representation of b by vectors $a_0, \ldots a_n$. The minimal value of t, $\underline{t}$ is given by*

$$\underline{t} = \frac{b_n(K_n - K_{n-1})}{b_{n-1} - b_n} \tag{6}$$

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FORMATION OF MASSLESS BOSE PARTICLES WITH SPINS 1 AS A RESULT OF RANDOM FLUCTUATIONS OF VACUUM FIELDS

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Fluctuations of quantum vacuum fields are a fundamental property of nature. Since the energy of the vacuum is an essentially greater part of the energy of the universe, then, obviously, its research is an actual problem of modern theoretical and mathematical physics. Note that quintessence (dark energy) and cosmic acceleration are often discussed in the framework of various approaches describing the quantum vacuum (QV), which necessarily include scalar fields. Recall that the properties of a quantum vacuum fields (QVF) can be studied within the framework of quantum field theory (QFT), ie quantum electrodynamics and quantum chromodynamics. Note, that QFT could accurately describe QV if it were possible to exactly summarize the infinite series of perturbation theories, that is typical of field theories. However, it is well-known that the perturbation theory for QFT breaks down at low energies (for example, QCD or the theory of superconductivity) field operators may have non-vanishing vacuum expectation values called *condensates*. Moreover, in the Standard Model precisely the non-zero vacuum expectation value of the Higgs field, arising from spontaneous symmetry breaking, is the principled mechanism allowing to acquire masses of other fields of theory. To overcome these difficulties and to conduct a consistent and comprehensive study of the QVF, we developed a nonperturbative approach based on a system of complex stochastic equations of the Langevin-Weyl type describing the motion of a massless particle with spin 1.

Definition. Let us consider the following stochastic differential equations (SDE):

$$\partial_t \boldsymbol{\psi}^{\pm}(\mathbf{r},t) \mp c\big(\mathbf{S}\cdot\boldsymbol{\nabla}\big)\boldsymbol{\psi}^{\pm}(\mathbf{r},t)=0, \quad \nabla\cdot\boldsymbol{\psi}^{\pm}(\mathbf{r},t)=0, \tag{1}$$

where c is the field propagation velocity, which differs from the velocity of light c_0 in vacuum, $\boldsymbol{\psi}^{\pm}(\mathbf{r},t)$ denote a random wave functions describing, the particle with the spin projection +1 and -1, respectively, and $\mathbf{S}=(S_x,S_y,S_z)$ denotes the set of matrices:

$$S_x=\begin{bmatrix}0&0&0\\0&0&-i\\0&i&0\end{bmatrix},\ S_y=\begin{bmatrix}0&0&i\\0&0&0\\-i&0&0\end{bmatrix},\ S_z=\begin{bmatrix}0&-i&0\\i&0&0\\0&0&0\end{bmatrix}. \tag{2}$$

Theorem. *If the QVF obeys the Langevin-Weil SDE (1)-(2), then massless Bose particles with spin 1 can form in the statistical equilibrium limit.*

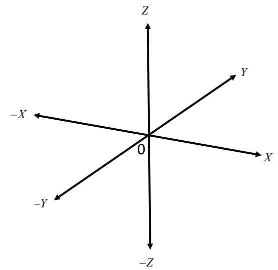

Figure 1: The coordinate system $\{X, Y, Z\}$ divides the 3D space into eight spatial regions by help three planes. The boson of a vector field with projection of spin +1 is a 2D- structure consisting of six components localized on the following planes $\phi_x^+[(-Y, Z) \cup (Y, -Z)]$, $\phi_y^+[(-X, Z) \cup (X, -Z)]$ and $\phi_z^+[(-X, Y) \cup (X, -Y)]$, respectively.

The latter means that the vector of the Hilbert space on a finite time interval τ (the relaxation time) self-averaging and independent of time:

$$\langle \boldsymbol{\psi}^{\pm}(\mathbf{r}, t)\rangle_\tau = \begin{bmatrix} \phi_x^{\pm}(\mathbf{r}) \\ \phi_y^{\pm}(\mathbf{r}) \\ \phi_z^{\pm}(\mathbf{r}) \end{bmatrix}. \tag{3}$$

As for the projections of the wave vector, they consist of two terms $\phi_\sigma^{\pm}(\mathbf{r}) = \phi_\sigma^{\pm(r)}(\mathbf{r}) + i\phi_\sigma^{\pm(i)}(\mathbf{r})$, each of which has the form described by the wave function of a hydrogen-like atom localized on the corresponding plane (see Fig. 1):

$$\phi_x^{\pm(r,i)}(\mathbf{r}) = \Lambda_{nl}(\mathbf{r})Y_{l,m}(\theta, \varphi), \ \Lambda_{nl}(r) = \frac{(b)^{3/2}(br)^l e^{-br/2} L_{n-l-1}^{2l+1}(br)}{\sqrt{2n(n-l-1)!(n+l)!}}, \tag{4}$$

where $b = (2/na_p)$ and $L_{n-l-1}^{2l+1}(br)$ is the generalized Laguerre polynomials, $Y_{l,m}(\theta, \varphi)$ is a spherical function, the principal quantum number $n = 1, 2, ...$ and the integer $l \leq n - 1$. Note that the set of bosons with spin projections ± 1 form vector fields. It is possible also formation of bosons with zero-spin by entangling of two bosons, respectively, with spin projections +1 and -1. Such set of bosons form scalar field.

IS THE HAMILTONIAN MECHANICS AND IN GENERAL CLASSICAL MECHANICS REVERSIBLE?

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It is well known that the classical equations with respect to the evolution parameter -the time "t", are invertible. This fact allows us to formulate the Cauchy problem for studying motion of a system of bodies. However, as shown by numerous theoretical and numerical studies, starting with a three-body system, the dynamic problem is usually not integrable and, moreover, often exhibits chaotic behavior in significant areas of the phase space. The latter circumstance again raises the question of the irreversibility of classical mechanics as one of the most important problem in the theory of dynamical systems and, accordingly, of modern physics and mathematics [1].

Theorem. *If the total interaction potential between the particles depends only on their relative distances, then the Newtonian general three-body problem reduces to a system of the sixth order, in addition, the representation is irreversible with respect to the evolution parameter:*

$$\begin{aligned}
\dot{\xi}^1 &= a_1\big\{(\xi^1)^2-(\xi^2)^2-(\xi^3)^2-\Lambda^2\big\}+2\xi^1\big\{a_2\xi^2+a_3\xi^3\big\}, \quad \xi^1=\dot{x}^1,\\
\dot{\xi}^2 &= a_2\big\{(\xi^2)^2-(\xi^3)^2-(\xi^1)^2-\Lambda^2\big\}+2\xi^2\Big\{a_3\xi^3+a_1\xi^1\Big\}, \quad \xi^2=\dot{x}^2,\\
\dot{\xi}^3 &= a_3\big\{(\xi^3)^2-(\xi^1)^2-(\xi^2)^2-\Lambda^2\big\}+2\xi^3\big\{a_1\xi^1+a_2\xi^2\big\}, \quad \xi^3=\dot{x}^3, \qquad (1)
\end{aligned}$$

where $\dot{\xi}=d\xi/ds$ and s is the length of arc along of the geodesic curve (timing parameter).

Note that the system (1) is defined on the Riemannian manifold, $\mathcal{M}=\big[\{x\}=(x^1,x^2,x^3)\in\mathcal{M}_t; g_{ij}(\{x\})=\big(E-U(\{x\})\big)\delta_{ij}\big]$, where, $a_1(\{x\}),a_2(\{x\}),\ a_3(\{x\})$ and $\Lambda(\{x\})$ are a some regular functions of coordinates, E and $U(\{x\})$- full energy and interaction potential of the system, in addition, $ds=\sqrt{g_{ij}dx^idx^j}$.

The transformations between the set of Jacobi coordinates $\{\varrho\}$ and the local coordinate system $\{x\}$ is given in differential form:

$$\begin{aligned}
d\varrho_1 &= \mathrm{x}_1dx^1+\mathrm{x}_2dx^2+\mathrm{x}_3dx^3,\\
d\varrho_2 &= \mathrm{y}_1dx^1+\mathrm{y}_2dx^2+\mathrm{y}_3dx^3,\\
d\varrho_3 &= \mathrm{z}_1dx^1+\mathrm{z}_2dx^2+\mathrm{z}_3dx^3, \qquad (2)
\end{aligned}$$

where the sets $(\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3)$, $(\mathrm{y}_1, \mathrm{y}_2, \mathrm{y}_3)$ and $(\mathrm{z}_1, \mathrm{z}_2, \mathrm{z}_3)$ are solutions of an incomplete system of algebraic equations at the current point $\{\varrho_0\}$:

$$\begin{aligned}
\mathrm{x}_1^2 + \mathrm{y}_1^2 + \gamma^{33}(\varrho_0)\mathrm{z}_1^2 &= g(\{\varrho_0\}), \quad \mathrm{x}_1\mathrm{x}_2 + \mathrm{y}_1\mathrm{y}_2 + \gamma^{33}(\varrho_0)\mathrm{z}_1\mathrm{z}_2 = 0,\\
\mathrm{x}_2^2 + \mathrm{y}_2^2 + \gamma^{33}(\varrho_0)\mathrm{z}_2^2 &= g(\{\varrho_0\}), \quad \mathrm{x}_1\mathrm{x}_3 + \mathrm{y}_1\mathrm{y}_3 + \gamma^{33}(\varrho_0)\mathrm{z}_1\mathrm{z}_3 = 0,\\
\mathrm{x}_3^2 + \mathrm{y}_3^2 + \gamma^{33}(\varrho_0)\mathrm{z}_3^2 &= g(\{\varrho_0\}), \quad \mathrm{x}_2\mathrm{x}_3 + \mathrm{y}_2\mathrm{y}_3 + \gamma^{33}(\varrho_0)\mathrm{z}_2\mathrm{z}_3 = 0,
\end{aligned} \tag{3}$$

where $\gamma^{33}(\varrho_0)$ is a regular function that is exactly defined in the problem.
The proof of the theorem enables us to obtain the following reduced Hamiltonian:

$$\mathcal{H}\big(\{x\};\{\dot{x}\}\big) = \frac{1}{2}g(\{x\})\Big\{\sum_{i=1}^{3}\big(\dot{x}^i\big)^2 + \big[J/g(x)\big]^2\Big\},$$

where J - the full angular momentum of the system and $g(\{x\}) = g_{ii}(\{x\})$. The latter obviously leads to irreversibility in the system of equations (1).

Thus, with the example of the three-body problem, we proved that there is a hidden irreversibility in Hamiltonian mechanics, which is the main cause of the onset of dynamic chaos in the phase space.

Proposition. *Let the metric of Riemannian space $g(\{x\})$ undergoes to the random fluctuations (quantum vacuum fluctuations):*

$$Q_f : g(\{x\}) \mapsto g(\{x\}) + \eta(s), \tag{4}$$

where Q_f displays a random influences, while $\eta(s)$ is a random function satisfying the following conditions:

$$\langle\eta(s)\rangle = 0, \qquad \langle\eta(s)\eta(s')\rangle = 2\varepsilon\delta(s - s'), \tag{5}$$

ε is the power of fluctuations. Taking into account (4), the system of equations (1) may be transformed to the system of stochastic differential equations using which for the joint probability distribution of the quantum scattering can be found the following equation:

$$\frac{\partial P}{\partial s} = \sum_{i=1}^{3}\frac{\partial}{\partial\xi^i}\big(A^i P\big) + \sum_{i,j,\,l,\,k=1}^{3}\epsilon_{ij}\frac{\partial}{\partial\xi^l}\Big[B^{il}\frac{\partial}{\partial\xi^k}\big(B^{kj}P\big)\Big]. \tag{6}$$

where A^i and B^{ij} are regular functions.

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FINITE APPROXIMATIONS OF TOPOLOGICAL ALGEBRAIC STRUCTURES

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Two approaches to finite approximation of topological algebraic structures will be discussed in this talk. The first one was introduced in [1] in terms of nonstandard analysis. The second one was presented in [1] by means of model theory. We show the definition of approximation of the paper [1] translated in the language of model theory is stronger, then the definition of the paper [2]. For example, all locally compact fields are not approximable by the finite associative rings in the sense of [1], while algebraically closed ones are approximable in the sense of [2]. We also formulate in terms of nonstandard analysis a weaker definition of approximation of locally compact structures by finite ones, according to which the field R is approximable by finite associative rings. Reformulation of this definition in standard terms or in terms of model is much more complicated.

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SELF-SIMILAR GROUPS, AUTOMATIC SEQUENCES, AND UNITRIANGULAR REPRESENTATIONS

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In the talk I will speak about natural linear representations of self-similar groups over finite fields. If the group is generated by a finite automaton, then matrices of these representations are automatic (i.e. automatically generated). This shows a new relation between two separate notions of automaticity: groups generated by automata and automatic sequences (like the Morse-Thue sequence). If the group acts on the p-adic tree by p-adic automorphisms, then the corresponding linear representation is a representation by infinite triangular matrices with automatic diagonals. A special attention will be paid to the infinite 2-group of intermediate growth constructed by the speaker in 1980.

The talk is based on joint results of Y. Leonov, V. Nekrashevych, V. Suschansky and speaker.

HYPERIDENTITIES OF ASSOCIATIVITY IN SEMIGROUPS

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The present paper is devoted to the study of balanced $\{2,3\}$-hyperidentities of the length of four in invertible algebras and $\{3\}$-hyperidentities of associativity in semigroups.

The following second order formula is called hyperidentity:

$$\forall X_1, \ldots, X_m \forall x_1, \ldots, x_n \qquad (W_1 = W_2) \tag{1}$$

where $X_1, \ldots, X_m$ are the functional variables, and $x_1, \ldots, x_n$ are the object variables in the words (terms) W_1, W_2. Usually, a hyperidentity is specified without universal quantifiers of the prefix of the equality: $W_1 = W_2$. According to the definition, the hyperidentity $W_1 = W_2$ is said to be satisfied in the algebra (Q, Σ) if this equality holds when every functional variable X_i is replaced by any arbitrary operation of the corresponding arity from Σ and every object variable x_j is replaced by any arbitrary element from Q.

If the arities of the functional variables are: $|X_1| = n_1$, ..., $|X_m| = n_m$, then the hyperidentity $W_1 = W_2$ is called $\{n_1, \ldots, n_m\}$-hyperidentity.

A hyperidentity is balanced if each object variable of the hyperidentity occurs in both parts of the equality $W_1 = W_2$ only once. A balanced hyperidentity is called first sort hyperidentity, if the object variables on the left and right parts of the equality are ordered identically. The number of the object variables in a balanced hyperidentity is called length of this hyperidentity.

The algebra (Q, Σ) with the binary and ternary operations is called $\{2,3\}$-algebra. A $\{2,3\}$-algebra is called non-trivial, if the sets of its binary and ternary operations are not singleton.

The present paper aims at classifying of the balanced $\{2,3\}$-hyperidentities of length four in invertible algebras and the description of the invertible algebras in which these hyperidentities hold, as well as at the description of the semigroups that polynomially satisfy ternary associative hyperidentities.

The following main results will be presented in the talk.

1. The balanced first sort $\{2,3\}$-hyperidentities of length four in non-trivial invertible algebras are classified;
2. The invertible $\{2,3\}$-algebras with a binary group operation and with the balanced first sort 2, 3-hyperidentities of the length four are described;
3. The invertible $\{2,3\}$-algebras with ternary group operation and with the balanced

first sort $\{2,3\}$-hyperidentities of length four are described;
4. The classes of the semigroups, which polynomially satisfy the associative $\{3\}$-hyperidentities are described.

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DISTRIBUTIVE LATTICES WITH STRONG ENDOMORPHISM KERNEL PROPERTY AS DIRECT SUMS

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The concept of the strong endomorphism kernel property for an universal algebra has been introduced by Blyth, Silva in [1] as follows.

Let $\theta \in \mathrm{Con}(A)$ be a congruence on A. We say that a mapping $f : A \to A$ is *compatible* with θ if $a \equiv b(\theta)$ implies $f(a) \equiv f(b)(\theta)$. An endomorphism of A is called *strong*, if it is compatible with every congruence $\theta \in \mathrm{Con}(A)$.

An algebra A has the *strong endomorphism kernel property* (SEKP) if every congruence relation on A different from the universal congruence ι_A is the kernel of a strong endomorphism of A.

Let V be a variety. Let A_i, $i \in I$ be algebras from V such that they all have one element subalgebra and we have chosen (distinguished) elements $e_{A_i} \in A_i$ such that $\{e_{A_i}\}$ is one element subalgebra if A_i. We denote $supp(f) = \{i;\ f(i) \neq e_{A_i}\}$ for $f \in \prod(A_i, i \in I)$. Now let us consider the following subalgebra B of $\prod(A_i, i \in I)$: $B = \{f \in \prod(A_i, i \in I); supp(f) \text{ is finite}\}$. We shall denote it as $\sum((A_i, e_{A_i}); i \in I)$, a direct sum of algebras A_i with distinguished elements.

Unbounded distributive lattices which have strong endomorphism kernel property (SEKP) were fully characterized in [2] using Priestley duality. The characterisation is as follows.

Theorem 1. *Let L be an unbounded distributive lattice. Then L has SEKP if and only if L is locally finite and there exists $c \in L$ such that for every $x < c$ or $x > c$ intervals $[x, c]$ (if $x < c$) and $[c, x]$ (if $x > c$) are (finite) Boolean.*

We shall call elements c from (2) of this theorem boolean elements of L.

In this note we shall show that an unbounded distributive lattice L which has SEKP can be written as a product $L \cong A \times B \times C$, where

– A is a special sublattice of $\sum((C_3, a); i \in U)$ – of a direct sum of U copies of 3 element chain $C_3 = \{0, a, 1\}$, $0 < a < 1$, with distinguished element a

– B is $\sum((\{0, 1\}, 1); i \in V)$ – a direct sum of V copies of 2 element chain with distinguished element 1 (top element) and

– C is $\sum((\{0, 1\}, 0); i \in W)$ – a direct sum of W copies of 2 element chain with distinguished element 0 (bottom element)

for appropriate sets U, V, W (any of which can be empty).

Moreover, $B \times C$ is isomorphic to a (convex) sublattice consisting of all boolean elements of L. Also, each product of such three lattices (direct sums) is the unbounded distributive lattice which has SEPK.

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EXTREMAL LENGTH AND SOME APPLICATIONS IN TEICHMULLER THEORY AND HYPERBOLIC GEOMETRY

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Extremal length of families of curves is a conformal invariant introduced by Beurling and Ahlfors in 1950's which has been studied extensively since then. In this talk we will study the asymptotic behavior of moduli of certain degenerating families of curves, and will describe two consequences of these estimates in Teichmuller theory and hyperbolic geometry. The first application is a joint work with Saric, where we show that there is an open and dense set of geodesic rays in the universal Teichmuller space $T(D)$, i.e. the space of all normalized quasisymmetric mappings of the unit circle, which converge at infinity to points in the Thurston boundary of $T(D)$. The second application is a joint work with Basmajian and Saric, where we provide several novel sufficient conditions on an infinite Riemann surface X (in terms of the hyperbolic geometry of X) implying that X does not support a Green function.

ON THE USAGE OF LINES IN GC_n-SETS

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A node set $\mathcal{X}$, with $|\mathcal{X}| = \binom{n+2}{2}$, in the plane is called GC_n-set if each node possesses fundamental polynomial in form of a product of n linear factors. We say that a node uses the line $Ax + By + C = 0$ if $Ax + By + C$ divides the fundamental polynomial of the node. It is a simple fact that any used line, i.e., a line which is used by a node, passes through at least 2 nodes and through at most $n+1$ nodes of $\mathcal{X}$. A line is called k-node line if it passes through exactly k-nodes of $\mathcal{X}$. An $(n+1)$-node line is called a maximal line. The well-known conjecture of M. Gasca and J. I. Maeztu [1] states that every GC_n-set has a maximal line. Until now the conjecture has been proved only for the cases $n \le 5$ [2]. Here we adjust and prove a conjecture proposed in [3]. Namely, by assuming that the Gasca-Maeztu conjecture is true, we prove that for any GC_n-set $\mathcal{X}$ and any k-node line ℓ the following statements hold:

- The line ℓ is not used at all, or it is used by exactly $\binom{s}{2}$ nodes of $\mathcal{X}$, where s satisfies the condition $\sigma := 2k - n - 1 \le s \le k$.
- If in addition $\sigma \ge 3$ then the line ℓ is necessarily a used line.

At the end, for each n and k with $\sigma = 2$ we bring an example of a GC_n-set and a nonused k-node line.

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COMPUTATION OF THE MOORE-PENROSE INVERSE FOR BIDIAGONAL MATRICES

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The Moore-Penrose inverse is the most popular type of matrix generalized inverses which has many applications both in matrix theory and numerical linear algebra. It is well known that the Moore-Penrose inverse can be found via singular value decomposition. In this regard, there is the most effective algorithm which consists of two stages. In the first stage, with the help of the Householder reflections, an initial matrix is reduced to the upper bidiagonal form (the Golub-Kahan bidiagonalization algorithm). The second stage is known in scientific literature as the Golub-Reinsch algorithm. This is an iterative procedure which with the help of the Givens rotations generates a sequence of bidiagonal matrices converging to a diagonal form. Acting in this way, an iterative approximation to the singular value decomposition of the bidiagonal matrix is obtained.

The principal intention of the report is to develop a method which can be considered as an alternative to the Golub-Reinsch iterative algorithm. Realizing the approach proposed in the study, the following two main results have been achieved. First, we obtain explicit expressions for the entries of the Moore-Penrose inverse of upper bidiagonal matrices. Secondly, based on the closed form formulas, we get a finite recursive numerical algorithm of optimal order of computational complexity. Thus, we can compute the Moore-Penrose inverse of an upper bidiagonal matrix without using the singular value decomposition.

MULTIPLE HYPOTHESES OPTIMAL TESTING WITH REJECTION OPTION FOR MANY OBJECTS

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The multiple statistical hypotheses testing with possibility of rejecting of decision for discrete independent observations is investigated for models consisting of two independent objects. The matrix of optimal asymptotical interdependencies of possible pairs of the error probability exponents (reliabilities) is studied.

For an asymptotically optimal test the probability of error decreases exponentially when the number of observations tends to infinity. Such tests were profoundly studied for case of two hypotheses by many authors. The sequence of such tests was called logarithmically asymptotically optimal (LAO). Haroutunian [1, 2] investigated the problem of LAO testing for multiple hypotheses.

In publications [3]– [5] many hypotheses LAO testing for the model consisting of many independent objects was studied. The multiple hypotheses testing problem with possibility of rejection of decision for arbitrarily varying object with side information was examined in [6] and [7]. This report is devoted to study of characteristics of logarithmically asymptotically optimal (LAO) hypotheses testing with possibility of rejection of decision for the model consisting of two independent objects. In the report two models are studied, the first when the rejection of decision is allowed to one of the objects and the second when the rejection of decision is allowed to both objects.

The study is based on information theoretic methods. Applications of information theory in mathematical statistics, specifically in hypotheses testing, are exposed in multiple works and also in the monographs by Cover and Thomas [8], Csiszár and Shields [9], Csiszár and Körner [10], Blahut [11].

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DIVERGENCE MEASURES FOR COMMUNITY DETECTION EVALUATION

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Community detection is a research area from network science dealing with the investigation of complex networks such as biological, social, computer networks, aiming to identify subgroupings (communities) of entities (nodes) that are more closely related to each other than with remaining entities in the network. Various community detection algorithms are used in the literature to obtain the community structure [1]. However the evaluation of the algorithms or their derived community structure is a very complicated task due to varying results on different networks. In searching good community detection algorithms the various comparison measures are used actively [2]. Information theoretic measures form a fundamental class and have recently received increasing interest [3].

In this paper we propose to use some f-divergence measures for community detection evaluation which can serve as a good alternative to existing measures used in the literature. Experiments on various community detection algorithms show the sensitivity of these measures in the special cases.

When particular algorithm is implemented, to assess the quality of the partition, it must be compared with other partitions or with available ground truth. This can be done using several evaluation measures. Most similarity measures can be divided into three categories: measures based on pair counting, cluster matching and information theory. The information theoretic measures have been employed in the clustering literature because of their strong mathematical foundation and ability to detect non-linear similarities.

The mutual information is the most basic similarity measure. The **mutual information** between two discrete random variables X and Y with probability distributions P_X, P_Y and joint probability distribution P_{XY} is defined as:

$$I(X;Y) = \sum_{y\in Y}\sum_{x\in X} P_{XY}(x,y)\log\frac{P_{XY}(x,y)}{P_X(x)P_Y(y)}\,.$$

Considering X and Y as two network partitions, mutual information is viewed as an evaluation measure to compare distinct community structures. Measures such as normalized mutual information, normalized variation of information and normalized information distance are modified variants of mutual information that are broadly used in the literature because they satisfy the properties of metric and normalization [3]. However, recently the application of these measures has been argued because of several disadvantages in special cases, such as the large number of communities [4].

We consider the family of **f-divergence measures** used in information theory and statistics [5]. Let $f : (0, \infty) \to R$ be a convex function with $f(1) = 0$, and let P and Q be two probability distributions. The f-divergence from P to Q is defined by

$$D_f(P\|Q) = \sum_x Q(x) f\left(\frac{P(x)}{Q(x)}\right).$$

Among others f-divergences include the

- Kullback-Leibler divergence, where $f(t) = t\log(t)$,

$$D(P\|Q) = \sum_x P(x)\log\left(\frac{P(x)}{Q(x)}\right),$$

- Total variation distance, where $f(t) = |t-1|$,

$$V(P,Q) = \sum_x |P(x) - Q(t)|,$$

- Hellinger distance, where $f(t) = (\sqrt{t}-1)^2$,

$$H(P,Q) = \sum_x \left(\sqrt{P(x)} - \sqrt{Q(x)}\right)^2,$$

- Jeffrey divergence, where $f(t) = \frac{1}{2}(t-1)\log(t)$,

$$D_j(P\|Q) = \sum_x (P(x) - Q(x))\log\left(\frac{P(x)}{Q(x)}\right),$$

- Capacitory discrimination, where $f(t) = t\log(t) - (t+1)\log(1+t)2\log(2)$,

$$C(P,Q) = D\left(P\|\frac{P+Q}{2}\right) + D\left(Q\|\frac{P+Q}{2}\right).$$

First note that $I(X;Y) = D(P_{XY}\|P_X P_Y)$, which needs normalization. Except KullbackLeibler divergence all measures satisfy the metric and normalization properties. We claim that above mentioned measures of discrimination from P_{XY} to $P_X P_Y$ can serve as a good alternatives to existing measures of community detection. Experiments on various community detection algorithms show the sensitivity of these measures in the case of large number of communities.

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ON SPECIAL CLASS OF SUBMANIFOLDS IN PSEUDOEUCLIDEAN RASHEVSKY SPACE E_n^{2n}

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Rashevsky space is a $2n$ dimensional pseudoriemannian space with metrics of index n. The structure equations of the pseudoeuclidean Rashevsky space (the curvature tensor is vanishing) may be presented in the form [1]

$$d\omega^I = \omega^I_K \wedge \omega^K,\ d\omega_I = -\omega^K_I \wedge \omega_K,\ d\omega^I_K = \omega^I_P \wedge \omega^P_K, \quad I, K, P = 1, 2, \ldots, n,$$

where the linear differential forms ω^I_K known as secondary forms generally don't depend on basic forms ω^1, ω^2, ..., ω^n, ω_1, ω_2, ..., ω_n and each other, are defined on the bundle $T^{(2)}E^{2n}_n$ of the second order frames on E^{2n}_n.

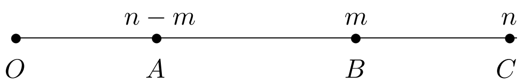

Figure. 1

The present work is devoted to the geometry of the special class of submanifolds M of dimension $2m$ $(2m > n)$ with structure of double fiber bundle, determined by the system of linear differential equations

$$\omega^{m+i} = \omega_i,\ \ \omega_{m+i} = -\omega^i, \qquad i = 1, 2, \ldots, n-m.$$

The bilinear form $d\varphi = \omega^I \wedge \omega_I$ playing the role of the metrics on M can be rewritten as follows: $d\varphi = \omega^I \wedge \omega_I = 2\omega^i \wedge \omega_i + \omega^\xi \wedge \omega_\xi$, $\xi = n-m+1, \ldots, m$. A special class of submanifolds $N \subset M \subset E^{2n}_n$, $\dim N = 2(2m-n)$ is determines by structure equations

$$\begin{aligned} &d\omega^\alpha = \omega^\alpha_b \wedge \omega^b,\ \ d\omega^a = 0,\ \ d\omega_\alpha = 0, \\ &d\omega_a = -\omega^\alpha_a \wedge \omega_\alpha,\ \ d\omega^\alpha_a = -C^{\alpha\beta}_i C^i_{ab}\omega^b \wedge \omega_\beta, \end{aligned} \tag{1}$$

where

$$\begin{aligned} &dC^i_{ab} = C^k_{ab}\omega^i_k + C^i_{\xi\eta\nu}\omega^\nu,\ \ dC^{\alpha\beta}_i = -C^{\xi\eta}_k\omega^k_i + C^{\xi\eta\nu}_i\omega_\nu, \\ &\text{rank } (C^i_{\xi\eta}) = u,\ \ \text{rank } (C^{\xi\eta}_i) = v, \\ &\alpha, \beta = n-m+1, \ldots, n-m+u; a, b = n-m+u+1, \ldots, m; u+v = 2m-n. \end{aligned} \tag{2}$$

Theorem 1. *Parametric equations of the submanifold N can be reduced to the following form: $X^i = x^i$, $X^\xi = x^\xi - C^\xi_i(y_{n-m+1}, \ldots, y_m)x^i$, $X^{m+i} = y_i$, $Y_i = y_i + C_i(y_{n-m+1}, \ldots, y_m)$, $Y_\xi = y_\xi$, $Y_{m+i} = -x^i$.*

Theorem 2. *Differential geometric structure (1), (2) is inducing on submanifold N by integral of the form*

$$\Omega = P(x)Q(y)\exp(x^\alpha y_\alpha + x^a y_a - \frac{1}{2}C_i^\alpha C_a^i x^a y_\alpha)\omega^{n-m+1}\wedge\cdots\wedge\omega^m,$$

where $P(x) = P(x^{n-m+1}, \dots, x^m)$ and $Q(y) = Q(y_{n-m+1}, \dots, y_m)$ are integrals of some positive smooth functions on x and y respectively.

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ABOUT SOME PROBLEMS FOR REGULAR DIFFERENTIAL OPERATORS

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We study the direct and inverse problems for the family of Sturm-Liouville operators, generated by a fixed potential q and the family of separated boundary conditions. We prove that the union of the spectra of all these operators can be represented as a smooth surface (as real analytic function of two variables), which has specific properties. From these properties we select those, which are sufficient for a function of two variables to be the union of the spectra of a family of Sturm-Liouville operators.

GENERALIZED HYPERGEOMETRIC SOLUTIONS OF THE HEUN EQUATIONS

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We present infinitely many solutions of the general Heun equation in terms of the generalized hypergeometric functions . Each solution assumes two restrictions imposed on the involved parameters: a characteristic exponent of a singularity should be a non-zero integer and the accessory parameter should obey a polynomial equation. Next, we show that the single confluent Heun equation with non-zero (this is the parameter characterizing the irregular singularity at the infinity) admits infinitely many solutions in terms of the generalized hypergeometric functions . For each of these solutions a characteristic exponent of a regular singularity of the confluent Heun equation is a non-zero integer and the accessory parameter obeys a polynomial equation. Each solution can be written as a linear combination with constant coefficients of a finite number of the Kummer confluent hypergeometric functions. Furthermore, we show that for the Ince limit the confluent Heun equation admits infinitely many solutions in terms of the functions . Here again a characteristic exponent of a regular singularity should be a non-zero integer and the accessory parameter should obey a polynomial equation. This time, each solution can be written as a linear combination with constant coefficients of a finite number of the Bessel functions. Finally, we present several applications of the listed solutions to the Schrdinger and Klein-Gordon equations, as well as to the quantum two-state dynamics.

ABOUT MEDIAL PAIRS OF CONTINUOUS AND STRICTLY MONOTONIC BINARY FUNCTIONS

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Theorem 1. *Let $M, N : I^2 \to I$ be binary operations which are medial pair, and let M and N be continuous and strictly monotonic for a both variables and M be medial operation. Then, N is also medial.*

Theorem 2. *Let $\circ, \cdot : I^2 \to I$ be binary operations which are medial pair, and let both operations be continuous, strictly monotonic for a both variables and pre-medial, and let operation $\circ$ be idempotent. Then operation $\circ$ is medial.*

CIRCULAR SLIDER GRAPHS

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De Bruijn graphs represent overlaps between consecutive subwords of the same length in a longer word. Under various names and in various guises they and their subgraphs currently enjoy a lot of popularity in mathematics (dynamical systems and combinatorics) as well as in the applications to computer science (data networks) and bioinformatics (DNA sequencing). At this talk I will present a new point of view on de Bruijn graphs and their subgraphs based on using circular words rather than linear ones.

CHANGING POINTS OF APN FUNCTIONS

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A construction in which a vectorial Boolean function $G : \mathbb{F}_{2^n} \to \mathbb{F}_{2^n}$ is constructed from a given $F : \mathbb{F}_{2^n} \to \mathbb{F}_{2^n}$ by changing its value at precisely one point of the underlying field is studied in [1] in the context of the open problem of the existence of Almost Perfect Nonlinear (APN) functions over $\mathbb{F}_{2^n}$ of algebraic degree n. Selecting a point $u \in \mathbb{F}_{2^n}$ to change and a nonzero value $v \in \mathbb{F}_{2^n}^*$, the function G can be written as $G(x) = F(x) + v(1 + (x+u)^{2^n-1})$ and satisfies $G(u) = F(u) + v$ and $G(x) = F(x)$ for all $x \neq u$. A number of characterizations of the properties of G and F are obtained in [1] and are used to derive non-existence results showing, for instance, that G cannot be APN if F is a power or plateaued function.

We study a more general construction in which a given function F over $\mathbb{F}_{2^n}$ is changed at several points. More precisely, given K distinct points $u_1, u_2, \ldots, u_K \subseteq \mathbb{F}_{2^n}$ of the field and K nonzero values $v_1, v_2, \ldots, v_K \subseteq \mathbb{F}_{2^n}^*$, we define G as

$$G(x) = F(x) + \sum_{i=1}^{K} v_i(1 + (x+u_i)^{2^n-1}). \tag{1}$$

We discuss different ways of characterizing the properties of G in terms of F, concentrating mostly on the possibility of obtaining an APN function from another APN function, and also examine the restriction of this general construction to some particular cases for which the problem of characterizing the relationship between the properties of F and G become more tractable.

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ABOUT ALGEBRAIC EQUATION WITH COEFFICIENTS FROM THE β-UNIFORM ALGEBRA $C_\beta(\Omega)$

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In the present work the algebraic equations of the following type

$$\lambda^n + a_1(x)\lambda^{n-1} + \cdots + a_n(x) = 0 \qquad (*)$$

are investigated, bounded and continuous functions given on a some locally compact Hausdorff space Ω. The aim of this work is to obtain the conditions which provide solvability of equation $(*)$ in the algebra of complex-valued, boundary and continuous functions on the space Ω. If we interested not with an individual equation $(*)$, but with a class of equations $(*)$, then the question about description of a locally compact space Ω, on which any equation of type $(*)$ are solvable be arised.

We note that for the compacts this problem sufficiently detailed were studied in the works (see [1]–[3]).

Let Ω be a locally compact Hausdorff space. We assume that the space Ω admits a "compact exhaustion, that is there exists a compacts $K_p \subset \Omega$, such that $K_p \subset K_{p+1}$ and $\Omega = \bigcup\limits_{p=1}^{\infty} K_p$. Recall that locally compact Ω is called a "hereditarily unicoherent if for any two connect closed subset $K_1, K_2 \subset \Omega$ their intersection is also a connect set. Simultaneously we note that for a connect finite latticed complex Ω (see [4]–[5]) the question about solvability on Ω an algebraic equations of type $(*)$ is connected with the fundamental group $\pi_1(\Omega)$, namely a group $H^1(\Omega;\mathbb{Z})$ is isomorphically to the group $\operatorname{Hom}(\pi_1(\Omega),\mathbb{Z})$. The class of all equations of type $(*)$ withuot multiple roots we denote $\mathfrak{A}_n(\Omega)$ as in [5] and $\overline{\mathfrak{A}_n(\Omega)} = \bigcup\limits_{k\leqslant n} \mathfrak{A}_k(\Omega)$.

In [5] it is shown that for a connect finite latticed complex Ω missing of a nontrivial homomorphism of a group $\pi_1(\Omega)$ in a Artins group of a "braid" $\mathcal{B}_n$ is equivalently to the fact that any equation $(*)$ without multiple roots is completely solvable, i.e. the belong to the class $\overline{\mathfrak{A}_n(\Omega)}$.

The main rersults of this work are the following approvals, which related to the topological algebras, more precisely to the β-uniform algebras (see [6]–[7]).

Theorem 1. *Let Ω be a locally connected, locally compact Hausdorff space, which admits a compact exhaustion and $\mathcal{A}(\Omega)$ is such β-uniform algebra on a space Ω, that for each $f \in \mathcal{A}(\Omega)$ there exists a natural number $k = k(f) \geqslant 2$ and $g \in \mathcal{A}(\Omega)$, such that $g^k = f$. Then $\mathcal{A}(\Omega) = C_\beta(\Omega)$.*

Theorem 2. *Let Ω be a connect, locally compact Hausdorff space admits a compact exhaustion. Then a β-uniform algebra $C_\beta(\Omega)$ will be algebraically closed if and only if when a space Ω is a locally connect and hereditarily unicoherent.*

Theorem 3. *Let Ω be a connect, locally compact Hausdorff space admits a connect compact exhaustion (i.e. $\Omega = \bigcup\limits_{p=1}^{\infty} K_p$, where K_p are a connect compacts). Supposed that for each K_p there exists sequence of inverse spectrum of a connect, finite latticed complexes $(K_{p,\alpha}; \omega_\alpha)$ converges to K_p, such that all $\pi_1(K_{p,\alpha}; \omega_\alpha)$ are commutative groups. Then necessary and sufficient condition for a complete solvability for all equations from the class $\overline{\mathfrak{A}_n(\Omega)}$ is a condition of division on $n!$ of group $H^1(\Omega; \mathbb{Z})$.*

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QUADRATIC FUNCTIONAL EQUATIONS ON QUASIGROUPS AND RELATED SYSTEMS

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It was almost sixty years ago that the seminal paper *Generalized associativity and bisymmetry on quasigroups,* Acta Math. Acad. Sci. Hungar. **11** (1960), 127–136; was published, where two important generalized quadratic functional equations on quasigroups

$$A_1(A_2(x,y),z) = A_3(x, A_4(y,z)), \tag{1}$$

$$A_1(A_2(x,y), A_3(u,v)) = A_4(A_5(x,u), A_6(y,v)) \tag{2}$$

were solved:

Theorem 1 (Aczél, Belousov, Hosszú)**.** *If four (six) quasigroups A_i $(i = 1, \ldots, 4$ $(i = 1, \ldots, 6))$ satisfy the equation of generalized associativity (1) (of generalized mediality (2)), then all A_i are isotopic to an (Abelian) group.*

The result was not just two important theorems, but also the explosive growth in the field. All three authors: János. Aczél [b. 1924], Valentin Danilovich Belousov [1925–1988] and Miklós Hosszú [1929–1980] become leading figures in the research concerning functional equations on quasigroups.

There are several approaches concerning generalizations of Theorem 1.

- Related to the form of equations (balanced i.e. permutational equations, quadritic equations, gemini equations, level equations, systems of equations etc.);
- Related to number of functions used (with one, two functions or generalized equations);
- Related to underlying algebra (binary, n–ary or infinitary quasigroups, division groupoids, 3–sorted quasigroups and GD–groupoids etc.);
- Related to methods of solving (using homomorphism of trees, graph methods);
- Related to underlying logic (1st order logic), hyperidentities (2nd order logic), equations in fuzzy context, equations in categorical context;
- Related to applications (in geometry (k–nets), in social sciences, in cryptography etc.).

We comment on some of these attempts. A particular emphasis is given to results by mathematicians from Serbia.

ON A CLASS OF EXTENSIONS BY COMPACT OPERATORS

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The report is devoted to the operator algebras $C^*_\varphi(X)$ in the case they are extensions of the algebra $C(S^1)$ of all continuous functions on the unit circle by compact operators.

The starting point is a selfmapping $\varphi : X \longrightarrow X$ on a countable set X with finite numbers of preimages of each point. This mapping generates a directed graph with vertices at the points of the set X and the edges $(x, \varphi(x))$. The algebra $C^*_\varphi(X)$ is generated by a composition operator

$$T_\varphi : l^2(X) \to l^2(X), \quad T_\varphi f = f \circ \varphi.$$

Theorem 1. *Let $C^*_\varphi(X)$ contains the algebra $K(l^2(X))$ of all compact operators on $l^2(X)$. Then the following are equivalent:*

1) $C^*_\varphi(X)$ is an extension of $C(S^1)$ by $K(l^2(X))$;

2) the Fredholm index of T_φ is finite.

If $\mathrm{index}(T_\varphi) = 1$ then $C^*_\varphi(X)$ is isomorphic to the Toeplitz algebra.

Let $\mathfrak{E}$ be the set of irreducible algebras $\{C^*_\varphi(X)\}_{\varphi\in\Phi}$ such that $\mathrm{index}(T_\varphi) \leq 0$.

Theorem 2. *Let $C^*_\varphi(X)$ and $C^*_\psi(X)$ are in $\mathfrak{E}$. Then they are isomorphic if and only if*

1) $\mathrm{index}(T_\varphi) = \mathrm{index}(T_\psi)$;

2) both mappings φ and ψ simultaneously admit (or not) the finite orbits.

We show that the set $\mathfrak{E}$ can be equipped with the semigroup structure isomorphic to $\mathbb{Z}_+$. We consider also different examples of nonisomorphic extensions of $C(S^1)$ by compact operators.

RESTRICTED SIMPLE LIE ALGEBRAS

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The theory of finite-dimensional Lie algebras over fields of positive characteristic $p > 0$ was initiated by E. Witt, N. Jacobson and H. Zasssenhaus. Sometime before 1937, Witt came up with an example of a simple Lie algebra of dimension p, which behaved completely differently from the known Lie algebras of characteristic zero. Over the thirty years following the discovery of the Witt algebra, several new families of simple modular Lie algebras were found and studied. In 1966 A. Kostrikin and I. Shafarevich introduced four families of simple finite-dimensional Lie algebras that covered all known simple non-classical Lie algebras. Thus algebras were the finite dimensional analogs of the infinite simple Lie algebras of Cartan, over the field of nonzero characteristic. They called these algebras Cartan type Lie algebras. Same time they conjectured that over an algebraically closed field of characteristic $p > 5$ a finite dimensional restricted simple Lie algebra is classical or Cartan type. In 1988 R. Block and R. Wilson proved that a finite dimensional restricted simple Lie algebra is classical or Cartan type which in part conforms the Kostrikin-Shafarevich conjecture. The Block-Wilson classification marked a major breakthrough in the theory and, also provided a framework for the classification of the nonrestricted simple Lie algebras. Finally, two decades after the Block-Wilson classification, A. Premet and H. Strade not only conformed the Kostrikin-Shafarevich original conjecture also completed classification simple modular Lie algebras over an algebraically closed field of characteristic $p > 3$. The final Block-Wilson-Strade-Premet Classification Theorem states: Every finite-dimensional simple Lie algebra over an algebraically close field of characteristic $p > 3$ is classical, Cartan, or Melikyan type (exists only in characteristic $p = 5$). The classification of simple restricted Lie algebras over an algebraically closed field of characteristic $p = 2$ or $p = 3$ is still open.

The aim of this talk is to give a comprehensive overview of Melikyan algebras, their realizations and brief summary of new results and open problems.

THE GIBBS PHENOMENON FOR STROMBERG SYSTEMS

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The Gibbs Phenomenon discovered by Henry Wilbraham in 1848 and rediscovered by Josiah Willard Gibbs in 1899, is the peculiar manner in which the Fourier series of some function behaves at a jump discontinuity. The n-th partial sum of the Fourier series has large oscillations near the jump, which might increase the maximum of the partial sum above that of the function itself. The overshoot does not die out as n increases, but approaches a finite limit.

Stromberg system is m-order spline system on $\mathbb{R}$, particularly, it is a modified Franklin system in the case $m = 0$. It was defined by Jan-Olov Stromberg in 1983 (see [1]). Stromberg system is obtained using Strombergs wavelet.

The Gibbs Phenomenon has been studied for Fourier series with respect to some famous systems. We studied the Gibbs phenomenon with respect to Stromberg systems (see [2]-[6]). We proved that the Gibbs phenomenon occurs for almost all points of $\mathbb{R}$.

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GEOMETRY OF A CLASS OF SEMISYMMETRIC SUBMANIFOLDS

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Let M be a Riemannian manifold with a Riemannian connection ∇, a curvature tensor R, a Ricci tensor R_1 and curvature operators $R(X,Y) = \nabla_X\nabla_Y - \nabla_Y\nabla_X - \nabla_{[X,Y]}$. If $R(X,Y)R = 0$, then Manifold M is called semi-symmetric, while if $R(X,Y)R_1 = 0$, it is called *Ricci*-semisymmetric. The implication $R(X,Y)R = 0 \Rightarrow R(X,Y)R_1 = 0$ is true. The local classification of Riemannian semisymmetric manifolds was obtained by Z. I. Szabo [1]. The basic structure theorem of *Ricci*-semisymmetric manifolds states that a smooth Riemannian manifold M satisfies the condition $R(X,Y)R_1 = 0$ if and only if it is either a two-dimensional, or an Einstein, or a semi-Einstein, or a direct product (locally) of the listed classes of manifolds [2]. Some classes of semi-Einstein submanifolds in Euclidean spaces were studied in [3], [4]. Herein, we give a geometric description of a semi-Einstein submanifold satisfying the condition $R(X,Y)R = 0$.

Theorem. *Suppose in an Euclidean space E_n an m-dimensional normally flat semi-Einstein submanifold M of nullity index $\mu \geq 1$ has at each point q $(3 \leq q \leq n-m+1)$ nonzero principal curvature vectors $n_1, \cdots, n_q$ with equal moduli and multiplicities $p_1 \geq 2, \cdots, p_q \geq 2$, respectively. If the eigendistributions $T^{(1,1)}, \cdots, T^{(1,q)}$, corresponding to these vectors, are parallel to each other on M (but not relative to the nullity distribution $T^{(0)}$), then*

(1) the vectors $n_1, \cdots, n_q$ form pairwise equal angles and $p_1 = \cdots = p_q$ $(= p)$,

(2) M satisfies the condition $R(X,Y)R = 0$, i.e. is semisymmetric, and locally represents a Cartesian product $E_{\mu-1} \times P$, where $E_{\mu-1}$ is plane of dimension $\mu - 1$, and submanifold P carries a $(q+1)$-component orthogonal conjugate system, consisting of q identical spheres $S_1^P(R), \cdots, S_q^P(R)$ and a straight line L, and represents a cone (with a generator L at each point) over a Cartesian product $S_1^P(R) \times \cdots \times S_q^P(R)$, which in its turn,

(a) is a $(pq+1)$-dimensional Einstein submanifold of Euclidean space $E_{n-\mu+1}$,

(b) belongs to the hypersphere $S^{n-\mu}(\bar{R})$ of space $E_{n-\mu+1}$; Radii R and $\bar{R}$are connected by a condition $\bar{R}^2 = q \cdot R^2$ and are linear (not constant) functions on L.

If the equality condition for the moduli of the vectors $n_1, \cdots, n_q$ is replaced by the condition of minimality of submanifold M, then (b) will be defined as follows: belongs to the hypersphere $S^{n-\mu}(\bar{R})$ of the space $E_{n-\mu+1}$ and is minimal in this hypersphere.

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RIGIDITY, GRAPHS AND HAUSDORFF DIMENSION

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We prove that if $E \subset \mathbb{R}^d$ is any compact set of Hausdorff dimension larger than $s_d(k) = d\frac{1}{k+1}$, then the m-dimensional Lebesgue measure of the set of congruence classes of $(k+1)$-point configurations of points from E is positive. This can be viewed as a generalization of the Falconer distance problem ([1]) on one hand, and of the Furstenberg-Katznelson-Weiss (see e.g. [2], [3]) type configuration results on the other. The proof relies on analytic, combinatorial and topological considerations.

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ON RECOVERING THE COMPOSITIONS OF TWO DISTRIBUTIONS FROM MOMENTS: SOME APPLICATIONS

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We study the problems of approximating and estimating the compositions of two functions (distributions). Namely, the models when the only available information about the underlying distributions represents the sequence of so-called transformed moments are considered. Several applications of proposed approximants in information theory and statistics are discussed. In particular, new moment-type approximates and estimates of the Shannen entropy, the Kullback-Leibler distance, as well as the quantile density function are derived, and their asymptotic properties are investigated. It is shown how the rate of approximations are related to the number of moments used in the proposed formulas. Finally, the modified versions of the approximants are introduced and the improvements of such versions (in terms of accuracy) are demonstrated by means of graphs and tables.

ESTIMATES FOR STRONG-SPARSE OPERATORS

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Let $\mathcal{S}$ be a sparse collection of dyadic intervals in R^d. Our interest is in weighted L^2 bound of the operator

$$S^*f = \sum_{B\in\mathcal{S}} \chi_B \cdot \sup_{A\supset B} \frac{1}{|A|}\int_A f.$$

It is trivial, that $S^*f \leq S(Mf)$ which gives $\|S^*\|_{L^2(w)\to L^2(w)} \leq [w]_{A_2}^2$. We prove, the sharp bound $\|S^*\|_{L^2(w)\to L^2(w)} \leq [w]_{A_2}^{3/2}$. The techniques are those of stopping cubes, Sawyer-type testing conditions and corona decomposition, in particular a localization method introduced by Lacey-Sawyer and Uriarte-Tuero.

AN EXTENSION OF ROMAN DOMINATING FUNCTION

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For a given graph $G = (V, E)$ with $V = V(G)$ and $E = E(G)$, a subset $S \subseteq V$ is a dominating set of G if every vertex $v \in V - S$ has a neighbour in S. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set in G, and a dominating set of G of cardinality $\gamma(G)$ is called a γ-set of G. A Roman dominating function on graph G is a function $f : V \to \{0, 1, 2\}$ such that if $v \in V_0$ for some $v \in V$, then there exist $w \in N(v)$ such that $w \in V_2$. The weight of a Roman dominating function is the sum $w_f = \sum_{v \in V(G)} f(v)$, and the minimum weight of w_f for every Roman dominating function f on G is called Roman domination number of G, denoted by $\gamma_R(G)$. The original study of Roman domination was motivated by the defense strategies used to defend the Roman Empire during the reign of Emperor Constantine the Great, 274-337 A.D. He decreed that for all cities in the Roman Empire, at most two legions should be stationed. Further, if a location having no legions was attacked, then it must be within the vicinity of at least one city at which two legions were stationed, so that one of the two legions could be sent to defend the attacked city. This part of history of the Roman Empire gave rise to the mathematical concept of Roman domination, as originally defined and discussed by I. Stewart, (1999) (Defend the Roman Empire!, Sci. Amer. 281 (6) (1999) 136-139) and C.S. ReVelle, K.E. Rosing, (2000) (Defendens imperium romanum: a classical problem in military strategy, Amer. Math. Monthly 107 (7) (2000) 585-594.)

Here we want to generalise the concept of Roman domination to a Roman $\{3\}$-domination that is defined as follows.

Definition 1. For a graph G, a Roman $\{3\}$-dominating function is a function $f : V \to \{0, 1, 2, 3\}$ having the property that for every vertex $u \in V$, if $f(u) \in \{0, 1\}$, then $f(N[u]) \geq 3$.

Here we may call the Roman $\{3\}$-dominating function with the name double Italian dominating function that is a generalization of Roman $\{2\}$-dominating function.

A Roman $\{3\}$-dominating function f relaxes the restriction that for every vertex $u \in V$, $f(N[u]) = \sum_{v \in N[u]} f(v) \geq 3$ maybe not necessarily the vertex u assigned with label 2. Note that for a Roman $\{3\}$-dominating function f, it is possible that $f(N[v]) = 2$ for some vertex with $f(v) = 2$. In terms of the double Roman Empire, this defence strategy requires that every location with no legion has at least

a neighbouring location with three legions, or at least one neighbouring location with two legions and one neighbouring location with one legion, or at least three neighbouring locations with one legion each, and every location with one legion has at least a neighbouring location with two legions or at least two neighbouring locations with one legion each.

We initiate the study of Roman $\{3\}$-domination and show its relationship to domination, Roman domination. Finally, we present an upper bound on the Roman $\{3\}$-domination number of a connected graph G in terms of the order of G and characterize the graphs attaining this bound.

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VARIETIES AND HYPERVARIETIES OF ALGEBRAS AND NEW DISCRETE MATHEMATICAL FUNCTIONS

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It is commonly known that the free Boolean algebra on n free generators is isomorphic to the Boolean algebra of Boolean functions of n variables. The free bounded distributive lattice on n free generators is isomorphic to the bounded lattice of monotone Boolean functions of n variables. In this talk we present the varieties and hypervarieties of algebras with similar functional representations of free finitely generated algebras.

A number of open problems are formulated.

INTERASSOCIATIVITY VIA HYPERIDENTITIES

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We study interassociativity of semigroups through the following hyperidentities of associativity ([1]-[3]):

$$X(Y(x,y),z) = Y(x,X(y,z)), \qquad (ass)_1$$

$$X(Y(x,y),z) = X(x,Y(y,z)), \qquad (ass)_2$$

$$X(X(x,y),z) = Y(x,Y(y,z)). \qquad (ass)_3$$

Moreover, in the q-algebras or e-algebras from $(ass)_3$ it follows $(ass)_2$ and from $(ass)_2$ it follows $(ass)_1$.

Definition. The semigroup $(S;\circ)$ is called $\{i,j\}$-interassociative to the semigroup $(S;\cdot)$ if algebra $S(\circ,\cdot)$ satisfies the hyperidentities $(ass)_i$ and $(ass)_j$, where $i,j = 1,2,3$. If $i = j$ the semigroup $(S;\circ)$ is called $\{i\}$-interassociative to $(S;\cdot)$.

We denote by $\mathrm{Int}_{\{i,j\}}(S;\cdot)$ the set of semigroups which are $\{i,j\}$-interassociative to semigroup $(S;\cdot)$. If $i = j$ the set $\mathrm{Int}_{\{i,j\}}(S;\cdot)$ is denoted by $\mathrm{Int}_{\{i\}}(S;\cdot)$.

Let $(\mathcal{F}(X);\cdot)$ be free semigroup generated by the set X, and $(\mathcal{FC}(X);\cdot)$ be the free commutative semigroup generated by the set X.

Theorem 1. $\mathrm{Int}_{\{1,2\}}(\mathcal{F}(X);\cdot) = \mathrm{Int}_{\{2\}}(\mathcal{F}(X);\cdot) = \{(\mathcal{F}(X);\cdot)\}$, *where* $|X| \geqslant 3$.

Theorem 2. $\mathrm{Int}_{\{3\}}(\mathcal{F}(X);\cdot) = \{(\mathcal{F}(X);\cdot)\}$.

Theorem 3. $\mathrm{Int}_{\{3\}}(\mathcal{FC}(X);\cdot) = \{(\mathcal{FC}(X);\cdot)\}$.

Theorem 4. $\mathrm{Int}_{\{2\}}(\mathcal{FC}(X);\cdot) = \{(\mathcal{FC}(X);*_x) \,|\, x \in \mathcal{FC}(X)\} \cup (\mathcal{FC}(X);\cdot)$, *where* $|X| \geqslant 4$, $a *_x b = axb$, $a,b \in \mathcal{FC}(X)$.

Theorem 5. *If* $|X| = 1$ *and* $X = \{a\}$, *then* $\mathrm{Int}_{\{1\}}(\mathcal{F}(X);\cdot) = \mathrm{Int}_{\{2\}}(\mathcal{F}(X);\cdot) = \{(\mathcal{F}(X);\cdot)\} \cup \{(\mathcal{F}(X);*_x) \,|\, x \in \mathcal{F}(X)\} \cup \{(\mathcal{F}(X);\circ)\}$, *where* $a^m \circ a^n = a^{m+n-1}$, $m,n \in \mathbb{N}$.

Using the result of [5] we prove the Theorem 1 for $|X| = 2$ streightforwardly.

In [4] is characterized $\mathrm{Int}_{\{1\}}(\mathcal{FC}(X);\cdot)$ and $\mathrm{Int}_{\{1,2\}}(\mathcal{FC}(X);\cdot)$. In [5] is considered $\mathrm{Int}_{\{1\}}(\mathcal{F}(X);\cdot)$.

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ARTINIAN AF C*-ALGEBRAS WHOSE MURRAY-VON NEUMANN ORDER OF PROJECTIONS IS A LATTICE

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Let A range over all AF C*-algebras whose Murray-von Neumann order of projections is a lattice. Then the Elliott involutive monoid of A will range over all countable MV-algebras. In fact, Elliotts classificationtheoremshows that Grothendieck's K_0 functor induces a one-one correspondence betweenthese two classes of structures. Using the spectral theory of MV-algebraswe study the Artinian property in these AF C*-algebras. For background on MV-algebras we refer to the present author's monograph *"Advanced ukasiewicz calculus and MV-algebras"*, Trends in Logic, Vol. 35, Springer, 2011.

THE INDEPENDENCE OF AXIOMS OF HYPERGROUP OVER GROUP

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The concept of hypergroup over group arises when one tries to extend the concept of quotient group in case of any subgroup of the given group. This concept was introduced in [1] and was developed in [2] and [3]. It generalizes and unifies the concepts of the group, of the field and of the linear space over field. In [3] the concept of (right) hypergroup over group is introduced as follows. Let H be an arbitrary group. A (right) hypergroup over group H is a set M together with a system of structural mappings $\Omega = (\Phi, \Psi, \Xi, \Lambda)$, where

- (Φ) $\Phi : M \times H \to M, \quad \Phi(a, \alpha) := a^{\alpha}$,
- (Ψ) $\Psi : M \times H \to H, \quad \Psi(a, \alpha) := {}^{a}\alpha$,
- (Ξ) $\Xi : M \times M \to M, \quad \Xi(a, b) := [a, b]$,
- (Λ) $\Lambda : M \times M \to H, \quad \Lambda(a, b) := (a, b)$

are mappings which satisfy following conditions:

P1) The mapping Ξ is a binary operation on M such that

(i) any equation $[x, a] = b$ with elements $a, b \in M$ has a unique solution in M;

(ii) (M, Ξ) has a left neutral element $o \in M$, i.e. $[o, a] = a$ for any element $a \in M$.

P2) The mapping Φ is an action of the group H on the set M, that is

(i) $(a^{\alpha})^{\beta} = a^{\alpha \cdot \beta}$ for any elements $\alpha, \beta \in H$ and for every $a \in M$;

(ii) $a^{\varepsilon} = a$ for each $a \in M$, where ε is the neutral element of the group H.

P3) For any element $\alpha \in H$, there exists an element $\beta \in H$ such that $\alpha = {}^{o}\beta$.

P4) The following identities (A1) - (A5) hold:

- (A1) ${}^{a}(\alpha \cdot \beta) = {}^{a}\alpha \cdot {}^{a^{\alpha}} \beta$,
- (A2) $[a, b]^{\alpha} = \left[a^{{}^{b}\alpha}, b^{\alpha}\right]$,

- (A3) $(a,b) \cdot {}^{[a,b]}\alpha = {}^{a}({}^{b}\alpha) \cdot (a^{{}^{b}\alpha}, b^{\alpha})$,
- (A4) $[[a,b],c] = [a^{(b,c)}, [b,c]]$,
- (A5) $(a,b) \cdot ([a,b],c) = {}^{a}(b,c) \cdot (a^{(b,c)}, [b,c])$.

We proved the following result.

Theorem 1. *The system* $\{(\mathrm{P1}), (\mathrm{P2}), (\mathrm{P3}), (\mathrm{A1}), (\mathrm{A2}), (\mathrm{A3}), (\mathrm{A4}), (\mathrm{A5})\}$ *of axisoms of hypergroup over group is independent.*

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ON ARITHMETICAL FUNCTIONS WITH INDETERMINATE VALUES OF ARGUMENTS

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In this paper the definition of arithmetical functions with indeterminate values of arguments is given. The notions of computability, strong computability and λ-definability for such functions are introduced. It is proved that every λ-definable arithmetical function with indeterminate values of arguments is monotonic and computable. It is proved that every computable, naturally extended arithmetical function with indeterminate values of arguments is λ-definable. It is also proved that there exist both λ-definable and non-λ-definable strong computable, monotonic, not naturally extended arithmetical functions with indeterminate values of arguments.

1. Arithmetical functions with indeterminate values of arguments. Let $M = N \cup \{\perp\}$, where $N = \{0, 1, 2, \ldots\}$ is the set of natural numbers, $\perp$ is the element which corresponds to indeterminate value. Let us introduce the partial ordering $\subseteq$ on the set . For every $m \in M$ we have: $\perp \subseteq m$ and $m \subseteq m$. A mapping $\varphi : M^\kappa \to M$, $k \geqslant 1$, is said to be arithmetical function with indeterminate values of arguments [1], [2].

Definition 1. *A function $\varphi : M^\kappa \to M$, $k \geqslant 1$, is said to be computable if there exists an algorithm [3], which for all $m_1, \ldots, m_k \in M$ stops with value $\varphi(m_1, \ldots, m_k)$ if $\varphi(m_1, \ldots, m_k) \neq \perp$, and stops with value $\perp$, or works infinitely if $\varphi(m_1, \ldots, m_k) = \perp$.*

Definition 2. *A function $\varphi : M^\kappa \to M$, $k \geqslant 1$, is said to be strong computable if there exists an algorithm [3], which stops with value $\varphi(m_1, \ldots, m_k)$ for all $m_1, \ldots, m_k \in M$.*

Definition 3. *A function $\varphi : M^\kappa \to M$, $k \geqslant 1$, is said to be monotonic if $(m_1, \ldots, m_k) \subseteq (\mu_1, \ldots, \mu_k)$ implies $\varphi(m_1, \ldots, m_k) \subseteq \varphi(\mu_1, \ldots, \mu_k)$ for all $m_i, \mu_i \in M$, $i = 1, \ldots, k$.*

Definition 4. *A function $\varphi : M^\kappa \to M$, $k \geqslant 1$, is said to be naturally extended if $\varphi(\ldots, \perp, \ldots) = \perp$.*

It is easy to see that every naturally extended function is monotonic.

2. On λ-definability of arithmetical functions with indeterminate values of arguments. Let us fix countable set of variables V and define the set of terms Λ [4].

1. if $x \in V$, then $x \in \Lambda$;
2. if $t_1, t_2 \in \Lambda$, then $(t_1 t_2 \in \Lambda)$;
3. if $x \in V$ and $t \in \Lambda$, then $(\lambda x t) \in \Lambda$.

We will use the abridged notation for terms: the term $(\ldots(t_1 t_2)\ldots, t_k)$, where $t_i \Lambda$, $i = 1, \ldots k$, $k > 1$, is denoted as $t_1 t_2 \ldots t_k$, and the term $(\lambda x_1(\lambda x_2(\ldots(\lambda x_n t)\ldots)$, where $x_j \in V$, $t \in \Lambda$, $j = 1, \ldots, n$, $n > 0$, is denoted as $\lambda x_1 x_2 \ldots x_n.t$.

The notion of a free and bound occurrence of a variable in a term and the notion of a free variable of a term are introduced in a conventional way. A term that does not contain free variables is said to be closed.

Terms t_1 and t_2 are said to be congruent (which is denoted as $t_1 \equiv t_2$) if one term can be obtained from the other by renaming bound variables. In what follows, congruent terms are considered identical.

The term obtained from a term t as a result of the simultaneous substitution of a term τ instead of all free occurrences of a variable x is denoted as $t[x := \tau]$. A substitution is said to be admissible if all free occurrences of variables of the term being substituted remain free after the substitution. We will consider only admissible substitutions.

Let us remind the notion of the β-reduction:

$$\beta = \{(\lambda x.t)\tau, t[x := \tau])|t, \tau \in \Lambda, x \in V\}.$$

A one-step β-reduction ($\rightarrow_\beta$), β-reduction ($\twoheadrightarrow_\beta$), and β-equality ($=_\beta$) are defined in a standard way.

We remind that the term $(\lambda x.t)\tau$ is referred to as β-redex. A term not containing β-redexes is referred to as β-normal form (further, simply normal form). The set of all normal forms is denoted by NF. A term t is said to have a normal form if there exists a term $t' \in NF$ such that $t =_\beta t'$. A term of the form $\lambda x_1 x_2 \ldots x_n.x t_1 t_2 \ldots t_k$, where $x, x_i \in V$, $t_j \in \Lambda$, $i = 1, \ldots, n$, $n \geqslant 0$, $j = 1, \ldots, k$, $k \geqslant 0$, is referred to us a head normal form. The set of all head normal forms is denoted by HNF. A term t is said to have a head normal form if there exists a term $t' \in HNF$ such that $t =_\beta t'$. It is known that $NF \subset HNF$, but $HNF \not\subset NF$.

We introduce the following notation for some terms: $I \equiv \lambda x.x$, $F \equiv \lambda xy.y$, $\Omega \equiv (\lambda x.xx)(\lambda x.xx)$, $\langle\perp\rangle \equiv \Omega$, $\langle 0\rangle \equiv I$, $\langle n+1\rangle \equiv \lambda x.xF\langle n\rangle$, where $x, y \in V$, $n \in N$. It is easy to see that: the term Ω does not have a head normal form, the term $\langle n\rangle$ is a closed normal form, and if $n_1 \neq n_2$, then $\langle n_1\rangle$ and $\langle n_2\rangle$ are not congruent terms, where $n, n_1, n_2 \in N$.

Definition 5. *A function $\varphi : M^\kappa \rightarrow M$, $k \geqslant 1$, is said to be λ-definable if there exists such term $\Phi \in \Lambda$, that for all $m_1, \ldots, m_k \in M$ we have:*
$\Phi\langle m_1\rangle \ldots \langle m_k\rangle =_\beta \langle\varphi(m_1, \ldots, m_k)\rangle$, *if* $\varphi(m_1, \ldots, m_k) \neq\perp$ *and*
$\Phi\langle m_1\rangle \ldots \langle m_k\rangle$ *does not have a head normal form, if* $\varphi(m_1, \ldots, m_k) =\perp$.

Theorem 1. *Every λ-definable arithmetical function with indeterminate values of arguments is monotonic and computable.*

Theorem 2. *Every computable, naturally extended arithmetical function with indeterminate values of arguments is λ-definable.*

Theorem 3. *There exist both λ-definable and non-λ-definable strong computable, monotonic, not naturally extended arithmetical functions with indeterminate values of arguments.*

MSC2010: 68Q01; 68Q05.

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ORIENTATION-DEPENDENT DISTRIBUTIONS OF CROSS-SECTIONS

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Let $R^n (n \geq 2)$ be the n-dimensional Euclidean space, $\mathbf{D} \subset R^n$ be a bounded convex body with inner points and V_n be n-dimensional Lebesgue measure in R^n. $C(\mathbf{D}, h) = V_n(\mathbf{D} \cap (\mathbf{D} + h)), \quad h \in R^n$, is called the covariogram of $\mathbf{D}$. Here $\mathbf{D} + h = \{x + h, x \in \mathbf{D}\}$. G. Matheron conjectured that the covariogram of a convex body $\mathbf{D}$ determines $\mathbf{D}$ within the class of all convex bodies, up to translations and reflections (see [1], [2]). Denote by S^{n-1} the $(n-1)$-dimensional sphere of radius 1 centered at the origin in R^n. We consider a random line which is parallel to $u \in S^{n-1}$ and intersects $\mathbf{D}$, that is a random line from the following set: $\Omega(u) = \{\text{lines which are parallel to } u \text{ and intersect } \mathbf{D}\}$. Let $\Pi r_{u^\perp}\mathbf{D}$ the orthogonal projection of $\mathbf{D}$ on the hyperplane $u^\perp$($u^\perp$ is the hyperplane with normal u and passing through the origin). A random line which is parallel to u and intersects $\mathbf{D}$ has an intersection point (denote by x) with $\Pi r_{u^\perp}\mathbf{D}$. We can identify the points of $\Pi r_{u^\perp}\mathbf{D}$ and the lines which intersect $\mathbf{D}$ and are parallel to u. The last means, that we can identify $\Omega(u)$ and $\Pi r_{u^\perp}\mathbf{D}$. Assuming that the intersection point x is uniformly distributed over the convex body $\Pi r_{u^\perp}\mathbf{D}$ we can define the following distribution function:

$$F(u, t) = \frac{V_{n-1}\{x \in \Pi r_{u^\perp}\mathbf{D} : V_1(g(u, x) \cap \mathbf{D}) < t)\}}{b_{\mathbf{D}}(u)}.$$

The function $F(u, t)$ is called orientation-dependent chord length distribution function of $\mathbf{D}$ in direction u at point $t \in R^1$, where $g(u, x)$ - is the line which is parallel to u and intersects $\Pi r_{u^\perp}\mathbf{D}$ at point x and $b_{\mathbf{D}}(u) = V_{n-1}(\Pi r_{u^\perp}\mathbf{D})$. We can introduce every vector $h \in R^n$ by $h = tu$, where u is the direction of h, and t is the length of h. Let $u \in S^{n-1}$ and $t > 0$ such that $\mathbf{D} \cap (\mathbf{D} + tu)$ contains inner points. Then $C(\mathbf{D}, u, t)$ is differentiable with respect to t and it holds that

$$-\frac{\partial C(\mathbf{D}, u, t)}{\partial t} = (1 - F(\mathbf{D}, u, t)) \cdot b(\mathbf{D}, u)$$

i.e. the problem of determining bounded convex domain by its covariogram is equivalent to that of determining it by its orientation dependent chord length distribution. In R^3 two types of orientation-dependent coross-section distributions can be considered. First is the probability that the random chord generated by intersection of the spatial line with the domain has length less than or equal to given number. In the second case random planes and their intersections with the domain are observed.

The main goal is to enlarge the class of domains for which the form of the orientation dependent chord length distribution function and the cross-section area distribution function are known (see [3]– [6]).

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ON ISOPERIMETRIC FUNCTIONS OF FINITELY PRESENTED GROUPS

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Let $G = \langle A \mid R \rangle$ be a group given by a finite generating set A and a finite set R of defining relators. Any word $r \in R$ vanishes in G. For any other trivial in G word w over the alphabet $A^{\pm 1}$, there is a derivation $w = w_0 \to w_1 \to \cdots \to w_{t-1} \to 1$, where 1 is the empty word and every w_i is obtained from w_{i-1} after one of the elementary transformations defined by R. The isoperimetric (or Dehn) function $D(n)$ of the finite presentation of G bounds from above the lengths t of the shortest derivations for all words of length at most n vanishing in G. Up to an asymptotic equivalence, $D(n)$ does not depend on the choice of a finite presentation for G. Therefore $D(n)$ is an asymptotic invariant of G measuring the complexity of the derivation of consequences from the defining relations. For example, $D(n)$ is linear, up to equivalence, iff the group G is word hyperbolic in terms of Gromov.

The speaker will recall known facts and present new results related to the behavior of Dehn functions, especially polynomially-bounded Dehn functions. As corollary, he obtains results on isoperimetric functions of universal covers for closed Riemannian manifolds.

CAN GEOMETRY BE REDUCED TO ALGEBRA?

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The question regarding a synthetic approach or an algebraic approach to geometry is an old one, with debates regarding the superiority of the synthetic approach over the algebraic one being carried on from Newton to Jakob Steiner. The question has faded from view in the 20th century with the clear victory of the algebraic camp. Axiomatic approaches themselves appeared to be most interested in arriving at some representation theorem, a statement linking all models of some axiom system to some known algebraic structure by means of some segment arithmetic or oherwise. Emil Artin's classic _Geometric Algebra_ does just that by way of configuration theorems such as Desargues's theorem.

In this talk, we shall examine what can be said from the vantage point of mathematical logic about the question whether geometry can be reduced to algebra. We will see, on the one hand, that this is possible only to a very limited extent, and will survey what is lost in the process. On the other hand, we will see by examining examples from reverse geometry, where one asks for minimal assumptions needed to prove a given theorem, that geometric thinking is more general and not reducible to algebra, for one notices that very weak axiom systems, that admit no representation theorem linking them to an algebraic structure, are often sufficient to prove interesting geometric statements.

DRINFELD-STUHLER MODULES

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Drinfeld-Stuhler modules are certain function field analogs of abelian varieties equipped with an action of a central simple algebra. The moduli spaces of these objects have played an important role in the proof of the local Langlands conjecture for $GL(n)$ in positive characteristic by Laumon, Rapoport and Stuhler.

We prove some basic results about Drinfeld-Stuhler modules and their endomorphism rings, and then examine the fields of moduli of these objects, with the goal of constructing examples of varieties over function fields violating the local-global principle.

A DISCRETE-TIME SIVS EPIDEMIC MODEL WITH CONSTANT POPULATION SIZE AND STANDARD INCIDENCE RATE

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The spread of infectious diseases in populations and how to control and eliminate them from the population is an important and necessary subject. Mathematical models are introduced to study what happens when an infection enters in a popution, and under which conditions the disease will be wiped out from population or persists in population.The literature about mathematical epidemic models that have been constructed and analysed for various type of diseases is very reach. Among these models the susceptible-infected-susceptible (SIS) epidemic models are one of the well known type of epidemic models. For the purpose of considering the effect of vaccination as a efficient strategy to control and aliminate infections, it is possible to add a compartment as the vaccination individuals to the SIS model and obtain the SIS epidemic model with vaccination namely, SIVS epidemic model. These models may be deterministic or stochastic, with constant or variable population size, and with standard or bilinear incidence rate. In this paper, we consider the following discrete-time SIS epidemic model with vaccination:

$$
\begin{aligned}
S_{t+1} &= (1-q)A - \beta S_t I_t / N_t + [1-(\mu+p)]S_t + \gamma I_t + \epsilon V_t, \\
I_{t+1} &= \beta S_t I_t / N_t + [1-(\mu+\gamma+\alpha)]I_t, \\
V_{t+1} &= qA + pS_t + [1-(\mu+\epsilon)]V_t.
\end{aligned}
$$

The susceptible individuals become infected at standard incidence rate $\beta S_t I_t / N_t$. The number of individuals N_t is variable in this model. But if we take $A = \mu N$ and $\alpha = 0$, then the population size will remain a constant value. Thus letting $V_t = N - S_t - I_t$, the coresponding difference equation is deleted and the following system of two difference equations is obtained:

$$
\begin{aligned}
S_{t+1} &= [(1-q)\mu + \epsilon]N - \beta S_t I_t / N + [1-(\mu+p+\epsilon)]S_t + (\gamma-\epsilon)I_t, \\
I_{t+1} &= \beta S_t I_t / N + [1-(\mu+\gamma)]I_t.
\end{aligned}
$$

We shall obtain some basic properties of this model such as: the equilibria and the basic reproduction number $\mathcal{R}_0$. Then stability of the equilibria is given with respect to $\mathcal{R}_0$ and moreover, the bifurcations of model are studied. Also, the results are challenged in some numerical examples.

RECIPROCITY LAWS AND ZETA-FUNCTIONS
(from Emil Artin to Robert Langlands)

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Emil Artin has made two fundamental contributions to algebraic number theory. He proved his version of the reciprocity law and introduced L-functions for non-abelian representations of the Galois groups of algebraic number fields. We give an overview of these results and show how the famous Langlands program developed from them.

DESCRIPTION OF THE BIOMETRIC IDENTIFICATION PROCESS OF TEETH WITH THE HELP OF COLORED PETRI NETS

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Biometric identification systems use given parameters and function on the basis of Colored Petri Nets as a modeling language developed for systems in which communication, synchronization and distributed resources play an important role. Colored Petri Nets combine the strengths of Classical Petri Nets with the power of a high-level programming language. Colored Petri Nets have both, formal intuitive and graphical demonstrate presentations. Graphical CPN model consists of a set of interacting modules which include a network of places, transitions and arcs. Mathematical representation has a well-defined syntax and semantics, as well as defines system behavioral properties.
One of the best known features used in biometry is the human finger print pattern. During the last decade other human features have become of interest, such as iris-based or face recognition. The objective of this paper is to introduce the fundamental concepts of Petri Nets in relation to tooth shape analysis.
Biometric identification systems functioning has two phases: data enrollment phase and identification phase. During the data enrollment phase images of teeth are added to database. This record contains enrollment data as a noisy version of the biometrical data corresponding to the individual. During the identification phase an unknown individual is observed again and is compared to the enrollment data in the database and then system estimates the individual.
Depending on given parameters and features teeth identification system is able to classify images for different application, among such biometric, dental or anthropological can be presented. Colored Petri Nets are best suited to analyze system functioning, error eliminating, validation and verification of biometric data.
In our research we use digital images of separate teeth obtained by means of photogrammetric methods, images of teeth obtained by dental arch 3D model segmentation and images of teeth obtained from segmented dental arch plaster models.

The purpose of modeling biometric identification system by means of Petri Nets is to reveal the following aspects of the functioning model:

- The efficiency of the model.

- Behavior of model.
- The existence of mistakes and accidents in the model.
- Simplify the model or substitute its separate components for more effective components without interfering system functioning.

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GAUSSIANITY TEST FOR MIXTURE COMPONENT DISTRIBUTION

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In this work we investigate a semiparametric testing approach to answer if the Gaussian assumption made by McLachlan *et al.* (2006) on the unknown component of their false discovery type mixture model was a posteriori correct or not. Based on a semiparametric estimation of the Euclidean parameters of the model (free from the Gaussian assumption), our method compares pairwise the Hermite coefficients of the model estimated directly from the data with the ones obtained by plugging the estimated parameters into the Gaussian version of the false discovery mixture model. These comparisons are incorporated into a sum of square type statistic which order is controlled by a penalization rule. We prove under mild conditions that our test statistic is asymptotically $\chi^2(1)$-distributed and study its behavior under different types of alternatives, including contiguous nonparametric alternatives. Several level and power studies are numerically conducted on models close to those considered in McLachlan *et al.* (2006) to validate the suitability of our approach. Finally we implement our testing procedure on the three microarray real datasets analyzed in McLachlan *et al.* (2006) and comment our results.

Keywords: Asymptotic normality, Chi-squared test, False Discovery Rate, nonparametric contiguous alternative, semiparametric estimator, two-component mixture model.

RIGID SOLVABLE GROUPS. ALGEBRAIC GEOMETRY AND MODEL THEORY

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A solvable group G is called rigid, more precisely m-rigid, if there exists a normal series of subgroups

$$G = G_1 > G_2 > \cdots > G_m > G_{m+1} = 1,$$

where all quotients G_i/G_{i+1} are abelian and when viewed as right modules over $\mathbb{Z}[G/G_i]$, do not have torsion. Free solvable groups and iterated wreath products of torsion free abelian groups are rigid, as well as their subgroups. A rigid group G is termed divisible if elements of the quotient G_i/G_{i+1} are divisible by non-zero elements of the ring $\mathbb{Z}[G/G_i]$, i.e. G_i/G_{i+1} is a vector space over the skew-field of fractions $Q(G/G_i)$ of the ring $\mathbb{Z}[G/G_i]$ (such a skew-field exists). We study an algebraic geometry over rigid groups and a model theory of divisible rigid groups.

BARYCENTRIC ALGEBRAS AND BEYOND

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Convex sets may be viewed as algebras equipped with a set of binary convex combinations that is indexed by the open unit interval I° of real numbers. Convex sets generate the variety $\mathcal{B}$ of barycentric algebras, which also includes semilattices where the semilattice multiplication is repeated uncountably many times.

Barycentric algebras provide a general algebraic framework for the study of convexity. They serve to model convexity and probability, allowing extensions of these important concepts to complex systems functioning at a number of different levels, and are used in hierarchical statistical mechanics for the study of complex systems [2, Ch. 9].

Barycentric algebras are defined by three types of hyperidentities for $p, q \in I^{\circ}$: the hyperidentity of *idempotence*

$$xx\,\underline{p} = x, \tag{1}$$

the hyperidentity of *skew-commutativity*

$$xy\,\underline{p} = yx\,\underline{1-p} =: yx\,\underline{p}', \tag{2}$$

and the hyperidentity of *skew-associativity*

$$[xy\,\underline{p}]\ z\,\underline{q} = x\ [yz\,\underline{q/(p\circ q)}]\ \underline{p\circ q}, \tag{3}$$

where $p \circ q = p + q - pq$. They belong to a broader class of (idempotent and entropic) algebras called modes, and an even broader class of distributive algebras [2, Chs. 5, 7].

Threshold barycentric algebras appeared when trying to answer a question concerning the axiomatization of convexity. Klaus Keimel had asked if the skew-associativity in the definition of barycentric algebras could simply be replaced by entropicity. It turned out that this is not possible. The first counter-example has grown into a family of algebras, called *threshold barycentric algebras*, where the open unit interval of operations is replaced by a possibly shorter subinterval that contains 1/2, keeping the remaining operations trivial. It was possible not only to answer Klaus Keimel's question, but also to show that each such shorter (non-trivial) subinterval generates all the operations of barycentric algebras. The threshold algebras have quite interesting properties, and provide a common framework for a whole spectrum of algebras, from usual barycentric algebras for threshold 0 to commutative binary modes (algebras with one binary commutative, idempotent and entropic operation) for threshold 1/2. They have already found some applications [3].

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ABOUT SOME BILINEAR FORMS ON THE LINEAR SPACES OF MATRICES

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Let $M^{n,n}$ means linear space of square matrices of order n.

In this talk bilinear symmetric forms $f(A, B)$ are defined for $A, B \in M^{n,n}$. Some of properties of these forms are proved. The obtained results concerning the main characteristics of matrices, such as trace, determinant, the coefficients of the characteristic polynomial of the matrix by the values of this forms.

ON SANDWICH SETS IN LEGAL SEMIGROUPS

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After introducing the notion of a sandwich set $S_l(e, f)$ for any idempotents e, f of a semigroup S belonging to any member of the legal class, it has been proved that $S_l(e, f)$ is a rectangular band. We also prove some results about legal sandwich sets of a semigroup S in any member of the legal class. Then we show that the subsets $S_l(e, f)f$ and $eS_l(e, f)$ of the semigroup S belonging to any member of the legal class, for any idempotents e, f in S, are subsemigroups of S, and the subsemigroups $S_l(e, f)$ and $S_l(e, f)f \times eS_l(e, f)$ of S are isomorphic.

Keywords: Legal semigroups, sandwich sets, isomorphisms.

IMPLICATION ZROUPOIDS: AN ABSTRACTION FROM DE MORGAN ALGEBRAS

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In 1934, Bernstein [1] gave a system of axioms for Boolean algebras in terms of implication only; however, his axioms were not equational. A quick look at his axioms would reveal that, with an additional constant, they could easily be translated into equational axioms. In 2012, this modified Bernstein's theorem was extended to De Morgan algebras in [10]. Indeed, it is proved in [10] that the varieties of De Morgan algebras, Kleene algebras, and Boolean algebras are term-equivalent to varieties whose defining axioms use only the implication $\to$ and the constant 0. Furthermore, a simplification of the (modified equational) axiom system of Bernstein is also given in [10].

These results motivated me to introduce a new (equational) class of algebras called "Implication zroupoids" in [10].

An algebra $\mathbf{A} = \langle A, \to, 0\rangle$, where $\to$ is binary and 0 is a constant, is called an *implication zroupoid* ($\mathbf{I}$-zroupoid, for short) if $\mathbf{A}$ satisfies:

(I) $\quad (x \to y) \to z \approx [(z' \to x) \to (y \to z)']'$, where $x' := x \to 0$;

(I_0) $\quad 0'' \approx 0$.

It turns out that the variety $\mathbf{I}$ of implication zroupoids contains, not only De Morgan algebras but also, the variety of $\vee$-semilattices with the least element 0. In fact, the structure of the lattice of subvarieties of $\mathbf{I}$ is very complex.

In [2]–[9], Juan Cornejo and I have obtained several results pertaining to the structure of the lattice of subvarieties of the variety of implication zroupoids.

In this talk I would like to survey some of our results on implication zroupoids and mention some new directions for future research.

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QUANTUM CALCULUS

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One of the main goals of the noncommutative geometry is to translate basic notions of topology, differential geometry and analysis into the language of Bahach algebras. In our talk we shall give several examples of such translation for the objects of classical analysis. Namely, we associate to real value function spaces, such as Sobolev space of half-differentiable functions or quasisymmetric homeomorphisms, some C^*-algebras of bounded operators in a Hilbert space. This correspondence was called by Alain Connes the quantum calculus.

ON BOL-MOUFANG TYPE IDENTITIES

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Standard information on groupoids, quasigroups and loops is given in [1, 2].

Identities that involve three variables, two of which appear once on both sides of the equation and one of which appears twice on both sides are called Bol-Moufang type identities.

Binary groupoid $(Q, \cdot)$ is called a quasigroup if for all ordered pairs $(a, b) \in Q^2$ there exist unique solutions $x, y \in Q$ to the equations $x \cdot a = b$ and $a \cdot y = b$.

In supposed talk we plan to speak about:

the number of groupoids of small orders with some Bol-Moufang type identities;

left (right) cancellation (division) groupoids with some Bol-Moufang type identities;

Bol-Moufang type identities defining commutative Moufang loops;

Bol-Moufang type identities which imply that corresponding groupoid (quasigroup) has a unit element;

Bol-Moufang type identities which imply that corresponding quasigroup has a non-trivial nucleus;

Bol-Moufang type identities and some inverse properties.

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SPECIALITY PROBLEM FOR MALCEV ALGEBRAS

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A Malcev algebra is an algebra that satisfies the identities

$$xx = 0, J(xy, z, x) = J(x, y, z)x,$$

where $J(x, y, z) = (xy)z+(yz)x+(zx)y$. Clearly, any Lie algebra is a Malcev algebra. If A is an alternative algebra then it forms a Malcev algebra A^- with respect to the commutator multiplication $[a, b] = ab - ba$. The most known examples of non-Lie Malcev algebras is the algebra O^- for an octonion algebra O and its subalgebra $sl(O)$ consisting of octonions with zero trace. Every simple non-Lie Malcev algebra is isomorphic to $sl(O)$.

The problem of speciality, formulated by A. I. Malcev in 1955, asks whether any Malcev algebra is isomorphic to a subalgebra of A^- for certain alternative algebra A. In other words, it asks whether an analogue of the celebrated Poincare-Bikhoff-Witt theorem is true for Malcev algebras. We show that the answer to this problem is negative, by constructing a Malcev algebra which is not embeddable into an algebra A^- for any alternative algebra A.

ARTIN'S INDUCTION THEOREM AND QUASIGROUP CHARACTERS

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A *quasigroup* $(Q, \cdot)$ is a set Q with a binary operation $\cdot$ of *multiplication*, such that for each element q of Q, the *left multiplication* $x \mapsto q \cdot x$ and *right multiplication* $x \mapsto x \cdot q$ are permutations of Q. In particular, groups are quasigroups. The *multiplication group* of a quasigroup $(Q, \cdot)$ is the subgroup of the full permutation group $Q!$ on Q that is generated by all the left and right multiplications. For example, the multiplication group of the quasigroup $(\mathbf{Z}/_n, -)$ of integers modulo n, under subtraction, is the dihedral group D_n of order $2n$.

If G is the multiplication group of a quasigroup Q of finite order n, consider the diagonal action $g\colon (x, y) \mapsto (xg, yg)$ of G on $Q \times Q$. The incidence matrices $I_n = A_1, \ldots, A_s$ of the orbits of G are a basis of a commutative algebra of complex matrices, which also has a basis $n^{-1}J_n = E_1, \ldots, E_s$ of orthogonal idempotents. (Here J_n is the all-ones $n \times n$-matrix.) Normalized versions of the change-of-basis matrices between these two bases yield the character table of the quasigroup Q, specializing to the usual character table when Q is a group [4]. For example,

$$\begin{array}{|ccc} \hline 1 & 1 & 1 \\ 1 & 1 & -1 \\ \sqrt{2} & -\sqrt{2} & 0 \end{array} \tag{1}$$

is the character table of $(\mathbf{Z}/_4, -)$.

Artin's Induction Theorem [1], stating that each character of a finite group is a rational linear combination of characters induced from cyclic subgroups, started as a tool to help express an L-function as a product of rational powers of abelian L-functions. We present a version of Artin's Induction Theorem for quasigroup characters [3], [4, §7.4]. Here, the scalars in the linear combinations are algebraic numbers. Indeed, the dimensions of quasigroup characters are algebraic integers in general. While this phenomenon is not yet fully understood, there appear to be some connections with quantum statistical dimensions. For example, the dimensions $1, 1, \sqrt{2}$ from the first column of (1) are the statistical dimensions of the physical representations of the conformal field theory for the scaling limit of the Ising model at the critical point [2, (1.57)].

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UNIVERSAL TOPOLOGICAL ABELIAN GROUPS

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Universal groups are defined for weakly linearly compact topological abelian groups. Some properties of these groups are studied.

For weakly linearly compact primary abelian groups with a distinct open and compact subgroup, we give the characteristic condition for its decomposition into a direct sum of groups of rank 1.

We present the following results.

Theorem 1. *All ρ-primary linearly compact groups are subgroups of the topological direct sum of groups of type $C(P^\infty)$.*

Corollary 1. *Any p-primary linearly discrete group is a factor group of a direct algebraic sum of an algebraic sum of groups of type Z_p.*

Corollary 2. *Any p-primary linearly discrete group is a closed subgroup of a direct algebraic sum of a countable set of groups of type Q_p and groups of type $C(P^\infty)$.*

Theorem 2. *A discrete Abelian group G admits a linearly compact topologization if and only if it is a complete direct sum of groups of types $C(P^n)$, P^∞, Z_p and Q_p.*

Theorem 3. *Any p-primary linearly compact group is a subgroup of the group G which has the form*

$$G = \sum_\alpha G_\alpha : H_\alpha,$$

where G_α are groups of type $C(P^\infty)$, and H_α are their subgroups of finite order.

Theorem 4. *Any p-primary linearly compact group is a factor group of the group G which has the form*

$$G = \sum_\alpha G_\alpha : H_\alpha,$$

where all G_α are groups of type Z_p and all H_α are their subgroups of finite index.

Theorem 5. *Any p-primary weakly linearly compact group is isomorphic to a closed subgroup of the group G which has the form*

$$G = \sum_\alpha G_\alpha : H_\alpha \dot{+} \prod_\beta G_\beta \dot{+} \sum_\theta G_\theta,$$

where G_α and G_β are groups of type $C(P^\infty)$, H_α are their subgroups of finite order and G_θ is a group of type Q_p.

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Keywords: compact group, group characters, weakly linearly compact topological abelian groups.

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ALGORITHM FOR SOLVING THE EQUATIONS $2^n \pm \alpha \cdot 2^m + \alpha^2 = x^2$

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The diophantine equations $2^n \pm 2^m + 1 = x^2$ have been completely solved in non-negative integers n, m and x in 2002. Later F. Luca showed that $\alpha^n + \alpha^m + 1 = x^2$ does not hold if α is an odd prime. Recently K. Gueth and L. Szalay investigated the equations $2^n - 3 \cdot 2^m + 9 = x^2$ and $2^n + 3 \cdot 2^m + 9 = x^2$, and gave all the solutions, in the second case with the condition $n \geq m$.

Assume now that α is a fixed odd prime such that 2 is a non-quadratic residue modulo α. We provide an algorithm for solving

$$2^n + \alpha \cdot 2^m + \alpha^2 = x^2 \text{ with } (n \geq m), \quad \text{and} \quad 2^n - \alpha \cdot 2^m + \alpha^2 = x^2.$$

Beside the infinite family $n = 2t$, $m = t + 1$, $x = 2^t \pm \alpha$, $t \in \mathbb{N}$ sometimes there exists sporadic solution(s), for example if $\alpha = 5$, then $(n, m, x) = (4, 0, 6)$ and $(6, 3, 7)$ both satisfy $2^n - 5 \cdot 2^m + 25 = x^2$. The main goal is exactly the determination of such sporadic solutions.

Further numerical examples, and experiences will also be published.

ON SYMMETRIC PRODUCT FINSLER SPACES

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Symmetric structures are defined on Affine and Riemannian spaces. Then special symmetries like Σ-symmetry [1], sigma symmetry [2] and weakly symmetry are defined. Some of them are extended to tangent bundles and product manifolds. Presently, symmetries are defined on Finsler spaces [3]. In this paper we study locally, globally and weakly symmetries on canonical product Finsler spaces and prove some theorem om them.

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Keywords and phrases: Product of Finsler spaces, symmetry Finsler space.

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ON MODULAR AND CANCELLABLE ELEMENTS OF THE LATTICE OF SEMIGROUP VARIETIES

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An element x of a lattice $\langle L; \vee, \wedge\rangle$ is called
modular if $(\forall y, z \in L)\ (y \leq z \rightarrow (x \vee y) \wedge z = (x \wedge z) \vee y)$,
cancellable if $(\forall y, z \in L)\ (x \vee y = x \vee z \,\&x \wedge y = x \vee z \rightarrow y = z)$.

It is easy to see that a cancellable element is a modular one. Modular elements of the lattice $\mathbb{SEM}$ of all semigroup varieties were examined in [2, 3, 4], while cancelable elements of this lattice were considered in [1]. In particular, commutative semigroup varieties that are modular elements of $\mathbb{SEM}$ were completely determined in [4, Theorem 3.1], and it is verified in [1, Theorem 1.1] that, within the class of commutative varieties, the properties to be modular and cancellable elements in $\mathbb{SEM}$ are equivalent. The objective of this work is to prove that this equivalence is false in slightly wider class, namely in the class of all varieties that satisfies a permutational identity of length 3, that is an identity of the form $x_1x_2x_3 = x_{1\pi}x_{2\pi}x_{3\pi}$ where π is a non-trivial permutation on the set $\{1, 2, 3\}$. The following assertion generalizes Theorem 3.1 of [4].

Theorem. *A semigroup variety* $\mathbf{V}$ *satisfying a permutational identity of length 3 is a modular element of the lattice* $\mathbb{SEM}$ *if and only if* $\mathbf{V} = \mathbf{M} \vee \mathbf{N}$ *where* $\mathbf{M}$ *is either the trivial variety or the variety of all semilattices, while the variety* $\mathbf{N}$ *satisfies one of the following identity systems: 1)* $xyz = zyx$, $x^2y = 0$*; 2)* $xyz = yzx$, $x^2y = 0$*; 3)* $xyz = yxz$, $xyzt = xzty$, $xy^2 = 0$*; 4)* $xyz = xzy$, $xyzt = yzxt$, $x^2y = 0$.

In particular, this theorem implies that the variety given by the identities $xyzt = xyx = x^2 = 0$, $x_1x_2x_3 = x_{1\pi}x_{2\pi}x_{3\pi}$ where π is a non-trivial permutation on the set $\{1, 2, 3\}$ is a modular element of the lattice $\mathbb{SEM}$. But we have proved that this variety is not a cancellable element of $\mathbb{SEM}$.

This is the joint work with D. V. Skokov. The work is partially supported by Russian Foundation for Basic Research (grant 17-01-00551) and by the Ministry of Education and Science of the Russian Federation (project 1.6018.2017/8.9).

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ASYMPTOTICAL PROPERTIES OF THE RANDOM WALKS ON THE DISCRETE GROUPS: ABSOLUTE AND POISSON–FURSTENBERG BOUNDARIES

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For any finitely generated group with fixed symmetric system of generators we can define Laplasian and so called dynamical Cayley graph which is a graph of trajectories of canonical Markov chain corresponding to the Laplacian. The general problem is to describe a set of all indecomposable Markov chains with the same cotransition probabilities. This set was defined by speaker and called “absolute”.

The Poisson boundary of the group is the quotient of the part of absolute, corresponding to harmonic functions, but absolute is more general notion.

In the several recent papers by Vershik and Malyutin we found absolute for commutative groups, for free groups, and for some nilpotent groups like Hieenberg groups.

LOCAL FINITENESS FOR GREEN'S RELATIONS IN SEMIGROUP VARIETIES

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A semigroup variety is said to be locally K-finite, where K stands for any of Green's relations H, R, L, D, or J, if every finitely generated semigroup in this variety has only finitely many K-classes. We characterize locally K-finite varieties of finite axiomatic rank in the language of "forbidden objects".

NEVANLINNA'S VALUE DISTRIBUTION THEORY AND ITS APPLICATIONS

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Recently, Nevanlinna theory has been utilized to study and derive new theory, new problems relating factorization (in the composite sense) and value sharing of meromorphic functions, as well as the problems relating to the existence and growth of meromorphic solutions of certain types of functional (including differential-difference) equations. In the talk, Nevanlinna theory will be briefly reviewed first, and then some results and related old or new open problems obtained or posed mainly by the speaker and his co-workers will be reported, for further investigations.

ON SUBALGEBRAS OF PROBABILITY DISTRIBUTIONS OVER FINITE RINGS WITH UNITY

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We construct sets of probability distributions over a finite ring with unity that are closed under application of ring addition and multiplication to independent random variables: i.e. the sum and product of independent random variables with distributions from the constructed set also belong to this set.

ASYMPTOTIC ESTIMATES OF THE NUMBER OF SOLUTIONS OF SYSTEMS OF EQUATIONS WITH DETERMINABLE PARTIAL BOOLEAN FUNCTIONS

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In this paper is investigated a class of systems of equations with determinable partial (not everywhere defined) Boolean functions. Determined the asymptotic estimate of the number of solutions of systems of equations for the "typical" case (the whole range of changes of the number of equations).

Many problems of discrete mathematics, including problems which are traditionally considered to be complex, lead to the solutions of the systems of Boolean equations of the form

$$\left\{ \begin{array}{l} f_i\left(x_1, \ldots, x_n\right) = 1 \\ i = 1, ..., l \end{array} \right. \tag{1}$$

or to the revealing of those conditions, under which the system (I) has a solution. In general problem of realizing whether the system (l) has a solution or not is NP-complete [1]. Therefore it is often necessary to consider special classes of the systems of equations, using their specificity, or explore a number of solutions for the "typical" case.

Let $\{M(n)\}_{n=1}^{\infty}$ is the collection of sets, such that $|M(n)| \to \infty$when$n \to \infty$, ($|M|$ is the power of the set$M$), and $M^s(n)$ is the subset of the all elements from $M(n)$, which have the property S. We say, that almost all the elements of the set $M(n)$ have the property$S$, if $\left|M^S(n)\right| / |M(n)| \to 1$, when $n \to \infty$.

We denote by $S_{n,l}$ the set of all the systems of the form (1), where $f_i\left(x_1, \ldots, x_n\right), i = 1, ..., l-$ pairwise different Boolean functions of variables $x_1, x_2, ..., x_n$. It is easy to see, that $|S_{n,l}| = C^l_{2^{2^n}}$.

Let $B = \{0, 1\}$,$B^n = \{\tilde{\alpha}/\tilde{\alpha} = (\alpha_1, \alpha_2, ..., \alpha_n), \alpha_i \in B, 1 \le i \le n\}$. The vector $\tilde{\alpha}_i = (\alpha_1, \alpha_2, ..., \alpha_n) \in B^n$is called a solution of (1), if

$$\left\{ \begin{array}{l} f_i(\alpha_1, \alpha_2, ..., \alpha_n) = 1 \\ i = 1, ..., l \end{array} \right.$$

We denote by $t(S)$ the number of the solutions of the system S. In [2,3] it is shown the asymptotics of the number of the solutions $t(S)$ for almost all the systems S of the set $S_{n,l}$ the whole range of parameter l changes, when $n \to \infty$.

In this work a class of systems of equations with determinable partial (not everywhere defined) Boolean functions is considered. Found the asymptotic behavior of the number of solutions of systems of equations for a "typical" case.

Partial Boolean function $f(x_1, \ldots, x_n)$ on the vector $\tilde{\alpha} = (\alpha_1, \alpha_2, \ldots.., \alpha_n) \in B^n$ or is not defined, or is 0 or 1. Let $Q(n)$ denote the set of all partial Boolean functions, depending on variables $x_1, x_2, \ldots, x_n$. Obviously, $|Q(n)| = 3^{2^n}$. Let $R(n, l)$ denote the set of all systems of l equations of the form (1), where $f_i(x_1, \ldots, x_n), i = 1, \ldots, l$ are pairwise differing partial Boolean functions of the variables $x_1, x_2, \ldots, x_n$ ($f_i \neq f_j$ if $i \neq j$condition persists). It is easy to see, that $|R_{n,l}| = C^l_{3^{2^n}}$. The vector $\tilde{\alpha} = (\alpha_1, \alpha_2, \ldots.., \alpha_n) \in B^n$is called a solution of (1), if
$\begin{cases} f_i(\alpha_1, \alpha_2, \ldots.., \alpha_n) \neq 0 \\ i = 1, \ldots, l \end{cases}$ and at least one of the functions $f_i(x_1, \ldots, x_n), i = 1, \ldots, l$occurs $f_i(\alpha_1, \alpha_2, \ldots.., \alpha_n) = 1$. In other words, allowed the not defined on a vector $\tilde{\alpha}$ partial function redefine by the 1.

For the numbers of the solutions $t(S)$of almost all the systems S of the set $R(n, l)$ the following statement is true (here and further $f(n) \sim g(n)$, if $f(n)/g(n) \to 1$ when $n \to \infty$, $f(n) = o(g(n))$ if $f(n)/g(n) \to 0$ when $n \to \infty$. Everywhere under the log refers to the logarithm to the base 2).

Theorem.

1. *If $n - \ell(\log 3 - 1) \to \infty$ when $n \to \infty$, then for almost all the systems $S$of the set $R(n, l)$ occurs $t(S) \sim 2^n(2^l - 1)3^{-l}$.*

2. *If $n - \ell(\log 3 - 1) \to -\infty$ when $n \to \infty$, then almost all the systems $S$of the set $R(n, l)$ have no solutions.*

3. *If $n - \ell(\log 3 - 1)$ is restricted when $n \to \infty$, then for almost all the systems of the set $R(n, l, m)$ the number of the solutions $t(S)$ is restricted from above by an arbitrary function $\varphi(n)$, satisfying the condition $\varphi(n) \to \infty$, when $n \to \infty$.*

MSC2010: 06E30, 94C10.

Keywords: Boolean equations, solution of equation, partial boolean functions.

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COUNTING SYMMETRIC BRACELETS

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An r-ary bracelet of length n is an equivalence class of r-colorings of vertices of a regular n-gon, taking all rotations and reflections as equivalent. A bracelet is symmetric if a corresponding coloring is invariant under some reflection. We show that the number of symmetric r-ary bracelets of length n is $\frac{1}{2}(r+1)r^{\frac{n}{2}}$ if n is even, and $r^{\frac{n+1}{2}}$ if n is odd [1,2].

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COUNTING RAY CLASS CHARACTERS AND THE ARTIN PRIMITIVE ROOT CONJECTURE

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We present estimates on certain sums which are related both to Artin's primitive root conjecture and to counting certain Artin representations.

GROUPS SATISFYING POLYNOMIAL IDENTITIES

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We will discuss the evolving subject of prounipotent and pro-p groups satisfying an identity.

ON FREE k-NILPOTENT n-TUPLE SEMIGROUPS

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Following [1], a nonempty set G equipped with n binary operations $\boxed{1}, \boxed{2}, ..., \boxed{n}$, satisfying the axioms $(x \boxed{r} y) \boxed{s} z = x \boxed{r} (y \boxed{s} z)$ for all $x, y, z \in G$ and $r, s \in \{1, 2, ..., n\}$, is called an n-tuple semigroup. An element 0 of an n-tuple semigroup $(G, \boxed{1}, \boxed{2}, ..., \boxed{n})$ will be called zero if $x * 0 = 0 = 0 * x$ for all $x \in G$ and $* \in \{\boxed{1}, \boxed{2}, ..., \boxed{n}\}$. An n-tuple semigroup $(G, \boxed{1}, \boxed{2}, ..., \boxed{n})$ with zero 0 will be called nilpotent if for some $m \in \mathbb{N}$ and any $x_i \in G$ with $1 \leq i \leq m+1$, and $*_j \in \{\boxed{1}, \boxed{2}, ..., \boxed{n}\}$ with $1 \leq j \leq m$,

$$x_1 *_1 x_2 *_2 \ldots *_m x_{m+1} = 0.$$

The least such m will be called the nilpotency index of $(G, \boxed{1}, \boxed{2}, ..., \boxed{n})$. For $k \in \mathbb{N}$ a nilpotent n-tuple semigroup of nilpotency index $\leq k$ is said to be k-nilpotent.

An n-tuple semigroup which is free in the variety of k-nilpotent n-tuple semigroups will be called a free k-nilpotent n-tuple semigroup. If ρ is a congruence on an n-tuple semigroup G' such that G'/ρ is a k-nilpotent n-tuple semigroup, we say that ρ is a k-nilpotent congruence.

Let X be an arbitrary nonempty set and ω an arbitrary word in the alphabet X. The length of ω will be denoted by l_ω. Fix $n \in \mathbb{N}$ and let $Y = \{y_1, y_2, ..., y_n\}$ be an arbitrary set consisting of n elements. Let further $F[X]$ be the free semigroup on X, $F^\theta[Y]$ the free monoid on Y and $\theta \in F^\theta[Y]$ the empty word. Fix $k \in \mathbb{N}$ and define n binary operations $\boxed{1}, \boxed{2}, ..., \boxed{n}$ on

$$XY_{[k]} = \{(w, u) \in F[X] \times F^\theta[Y] \,|\, l_w - l_u = 1, \, l_w \leq k\} \cup \{0\} \quad \text{by}$$

$$(w_1, u_1) \boxed{i} (w_2, u_2) = \begin{cases} (w_1 w_2, u_1 y_i u_2), & l_{w_1 w_2} \leq k, \\ 0, & l_{w_1 w_2} > k, \end{cases}$$

$$(w_1, u_1) \boxed{i} 0 = 0 \boxed{i} (w_1, u_1) = 0 \boxed{i} 0 = 0$$

for all $(w_1, u_1), (w_2, u_2) \in XY_{[k]} \backslash \{0\}$ and $i \in \{1, 2, ..., n\}$. The algebra obtained in this way will be denoted by $FN_n^k S(X)$.

Theorem. *$FN_n^k S(X)$ is the free k-nilpotent n-tuple semigroup.*

Corollary. *The free k-nilpotent n-tuple semigroup $FN_n^k S(X)$ generated by a finite set $X \times \{\theta\}$ is finite. Specifically, $|FN_n^k S(X)| = \sum_{i=1}^{k} n^{i-1} |X|^i + 1$.*

We also consider separately one-generated free k-nilpotent n-tuple semigroups and describe the least k-nilpotent congruence on a free n-tuple semigroup [2].

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A UNIFORM STABILITY PRINCIPLE FOR DUAL LATTICES

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(joint work with Martin Vodička)

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We will present a highly uniform stability or "almost-near" theorem for dual lattices of vector lattices $L \subseteq \mathbb{R}^n$. More precisely, we show that, for a vector x from the linear span of a lattice $L \subseteq \mathbb{R}^n$ with the Minkowski's first successive minimum $\lambda_1(L) \geq \lambda > 0$ to be ε-close to some vector from the dual lattice $L^\star$ of L, it is enough that the euclidean inner products $u\,x$ are δ-close (with $\delta < 1/3$) to some integers for all vectors $u \in L$ satisfying $\|u\| \leq r$, where $r > 0$ depends on n, λ, δ and ε, only. The result is derived as a consequence of its nonstandard version, formulated in terms of finite elements and the equivalence relation of infinitesimal nearness on the nonstandard extension ${}^*\mathbb{R}^n$ of $\mathbb{R}^n$: If x is a finite vector from the internal linear span of an internal lattice $L \subseteq {}^*\mathbb{R}^n$, such that the inner product $u\,x$ is infinitesimally close to some integer for each finite vector $u \in L$, then x is already infinitesimally close to some vector y from the dual lattice $L^\star$. The results generalize earlier analogous results proved for integral vector lattices by M. Mačaj and the author in [1].

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