

SOUTH EAST ASIAN MATHEMATICAL SOCIETY

FINAL REPORT

Dynamical Systems and Bifurcation Analysis (DySBA)

Penang, Malaysia 6th-13th August 2018

Organised by

School of Mathematical Sciences, Universiti Sains Malaysia, 11800 USM, Malaysia.

with the support of

South East Asian Mathematical Society (SEAMS) Centre International de Mathématiques Pures et Appliquées (CIMPA) Commission for Developing Countries (CDC) International Mathematical Union (IMU) Universiti Sains Malaysia (USM) Division of Research and Innovation, USM Department of Higher Education, Ministry of Education, Malaysia Malaysian Mathematical Sciences Society

2018

SEAMS School on Dynamical Systems and Bifurcation Analysis (DySBA) Penang, Malaysia 6th-13th August 2018

I. Summary

In this school, we discussed the state-of-the-art developments in the field of dynamical systems via planned lectures, contributed talks, and hands-on workshop for bifurcation analysis. This school has become a unique platform, which provides active researchers from various fields of mathematics the opportunity of discussing recent developments in theoretical and computational techniques for dynamical systems and their applications.

During the lectures, we covered various kinds of modelling frameworks and demonstrated how the techniques from dynamical systems and bifurcation theory been applied to these systems. Our speakers employed different deterministic systems such as ordinary-differential equations, delay-differential equations, fractional-differential equations, partial-differential equations, integro-differential equations and discrete-time models. Additionally, we emphasised on recent developments in stochastic models by discussing some topics in the individual-based modelling approach. We also highlighted the applications of dynamical systems and bifurcation analysis in different scientific areas such as ecology, epidemiology, physics and engineering fields. Theoretical lectures were complemented by practical sessions on bifurcation analysis through numerical continuation software e.g. XPPAUT and Auto.

A website was created to disseminate important information about the school to the attendees. We also designed an online feedback portal to facilitate the process of giving inputs and comments about the courses, talks and hands-on workshop sessions. An online drive, accessible directly from the website of the school, contains all the lecture notes, slides, scientific papers, lab manuals and the codes used during computer lab session.

Website: http://seamsdysba.usm.my/index.php

Feedback portal:

https://forms.office.com/Pages/ResponsePage.aspx?id=wCwTWsfjxUWwcq2OBrKj9o HMpus_FOhFopYEsjyLDrZUMjA3VDRYQUdFQkhZTFNTN1RDWjM1TTRaMy4u

Drive:

http://seamsdysba.usm.my/index.php/programme/programme-book

II. Scientific Objectives and Rationale for the School

For the year 2018, School of Mathematical Sciences (PPSM), Universiti Sains Malaysia (USM) has been given the mandate and trust to organise a research school after a thorough selection process conducted by SEAMS and CIMPA. Dynamical systems and bifurcation analysis have been selected as the main topics during this school. The organisation of the SEAMS School 2018 on Dynamical Systems and Bifurcation Analysis (DySBA) is in line with the goal of promoting PPSM to become an excellent research centre in mathematical modelling and dynamical systems. The SEAMS School 2018 on DySBA has been designed as part of series of mathematics study programs that aim to provide opportunity for advanced learning experience in the fields of dynamical systems, and to introduce research-based learning for advanced undergraduate, postgraduate students as well as young researchers.

The main scientific objective of this school is to form a research networking among ASEAN mathematicians to collaborate within the broader research community, as well as opening up new opportunities for researchers to link up and work together in the fields of dynamical systems and bifurcation analysis. The school is also intended to foster a joint scientific collaboration between PPSM USM and other regional institutions such as Vietnamese and Thailand universities.

In general, this school was the first ever SEAMS school concerning dynamical systems and bifurcation theory in Malaysia. The successful organisation of this school demonstrates the ability of PPSM USM to participate actively in research engagement activities and scientific collaboration at the regional and international levels. In conjunction with the SEAMS School 2018 on DySBA, PPSM USM has been offered to collaborate with the South East Asia Center of Unité de Modélisation Mathématique et Informatique des Systèmes Complexes (UMMISCO-France), Vietnam and Institut de Recherche pour le Développement (IRD French) to jointly organise a research workshop on complex systems modelling at the end of the year in Penang, Malaysia.



III. Organiser

Advisor

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Participant Presentation & Publication

Dr. Norasrizal Aswad Abdul Rahman

IV. Speakers

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V. Participants

The SEAMS School 2018 on DySBA hosted 48 participants, which were postgraduate students, early career researchers and lecturers from across the globe. Among these attendees, 24 were males and 24 were females. The school attracted foreign participants from different countries such as Vietnam, Indonesia, Philippines, Cambodia, Thailand, Malaysia, Egypt, Saudi Arabia, Kenya and Nigeria.

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VI. School Programmes

Day/ Hours	Aug 6 th Monday	Aug 7 th Tuesday	Aug 8 th Wednesday	Aug 9 th Thursday	Aug 10 th Friday	
8:45 - 9:45	Registration	C1	C2	C3	C4	
9:45 - 10:30	Opening		02	00		
10:30 - 10:45	Break	Break	Break	Break	Break	
10:45 - 12:30	C1	C2	C3	C4	C5	
12:30 - 14:00	Lunch	Lunch	Lunch	Lunch	Lunch	
14:00 - 15:45	C1	C2	C3	C4	C5	
15:45 - 16:15	Discussion	Discussion	Discussion	Discussion	Discussion	
16:15 - 16:30	Break	Break	Break	Break	Break	
16:30 - 18:00	C1	C2	C3	C4	C5	
18:00 - 18:45	P1	P2	P3	P4	P5	

The final schedule was as follows:

Day/ Hours	Aug 10 th Friday	Day/ Hours	Aug 11 th Saturday		Aug 12 th Sunday		Aug 13 th Monday
8:45 - 10:30	C4	8:45 - 10:30	C5			8:45 - 10:30	USM Walk
10:30 - 10:45	Break	10:30 - 10:45	Break			10:30 - 10:45	Break
10:45 - 11:15	P5	10:45 - 11:30	T1 Workshop 1		uo	10:45 - 11:05	Briefing by Deputy Dean
11:15 - 12:30	C5	11:30 - 13:00	(Auto)			11:05 - 11:50	T4
12:30 - 14:30	Lunch	13:00 - 14:00	Lunch		surs	11:50 - 12:45	Sharing Session 1
14:30 - 16:15	C5	14:00 - 15:30	Workshop 2 (XPPaut)		Exe	12:45 - 14:00	Lunch
16:15 - 16:30	Break	15:30 - 16:15	T2			14:00 - 16:00	Sharing Session 2
16:30 - 18:30	C5	16:15 - 16:30	Break			16:00 - 16:15	Break
18:30 - 18:45	Discussion	16:30 - 17:30	ТЗ			16:15 - 17:30	Discussion &
		20:00 - 22:30	Dinner] [Networking

Venue:

- All courses (C1-C5), contributed talks (T1-T4) and participants' presentation (P1-P5) were held in the Conference Room, School of Mathematical Sciences, USM.
- Dinner: BumbleDee's Cafe, USM.
- Workshop: Mathematical Lab I & II, School of Mathematical Sciences, USM.

Legends:

Main Courses

C1: Introduction to Delay Differential Equations with Applications - Assoc. Prof. Dr. Juancho Collera C2: Recent Research in Dynamical System and Related Topics - Prof. Dr. Salmi Md. Noorani

C3: Coupling Different Modelling Approaches in the Study of Competition Ecosystems

- Dr. Nguyen Ngoc Doanh

C4: Dynamical Systems Analysis for Fractional Differential Equations - Assoc. Prof. Dr. Doan Thai Son

C5: Dynamics and Bifurcations in Lotka-Volterra Type Systems

- Dr. Kie Van Ivanky Saputra

Contributed Talks

T1: Fish Population Dynamics with Harvesting and Toxicant Effects

- Dr. Hamizah Mohd Safuan

T2: Mathematical Modeling and Stability Analysis of Population Dynamics - *Dr. Auni Aslah Mat Daud*

T3: Stability Index for the Characterization of Riddled Basin in a Coupled Dynamical System

- Dr. Ummu Atiqah Mohd Roslan

T4: Dynamical Systems Analysis of Local and Non-Local Dispersal Models

- Dr. Mohd Hafiz Mohd

Participants' Presentations

P1: A Global Stability Analysis of a Tuberculosis Epidemic Model with Two Delays - Jay Michael R. Macalalag

P2: About the Correctness of Mathematical Models of Small Transverse Vibrations Style Source Strings with Arbitrary Mass Distribution

- Meach Mon

P3: Analysis of a Discrete-Time Fractional Order SIR Epidemic Model for Childhood Diseases

- Mahmoud Ahmed Mohamed Abdelaziz

P4: Dynamical Systems Analysis of a Fractional-Order Extended Rosenzweig-Macarthur Model with a Prey Refuge

- Elshahed Mahmoud Moustafa

P5: Nonautonomous Young Differential Equations Revisited

- Phan Thanh Hong

Hands-on Workshops

Workshop 1: Hands-on Workshop using Auto

- Dr. Kie Van Ivanky Saputra Workshop 2: Hands-on Workshop using XPPAUT

- Dr. Mohd Hafiz Mohd

Outline of the Main Courses and Contributed Talks:

Main Course C1: Introduction to Delay Differential Equations with Applications

Assoc. Prof. Dr. Juancho Collera

Abstract: This course is an introduction to delay differential equations (DDEs). The approach is rather intuitive and often pointing out the similarities and differences with ordinary differential equations (ODEs). The first half of the course will provide familiarity with the basic theories in DDEs including use of available software for solving systems of DDEs. The second half of the course focuses on some examples in theoretical ecology and will use the basic theories from the first half. We end by mentioning some related works and outlook.

Course Outline:

- 1. Introduction
 - 1.1. Examples of Delay Differential Equation Systems
 - 1.2. Delayed Negative Feedback
 - 1.3. Solving Delay Equations using a Computer
- 2. Linear Systems and Linearization
 - 2.1. Autonomous Linear Systems
 - 2.2. The Characteristic Equation
 - 2.3. Hopf Bifurcation Theorem
- 3. Predator-Prey Model with Time Delay and Constant Rate of Harvesting
 - 3.1. Local Stability Analysis
 - 3.2. Stability Switches via Hopf Bifurcations
 - 3.3. Numerical Simulations
- 4. Delayed Lotka-Volterra Intraguild Predation Model
 - 4.1. Local Stability Analysis
 - 4.2. Order-Three Quasi-Polynomials
 - 4.3. Related Works
 - 4.4. Some Outlook

Simulation Platforms: MATLAB and/or MAPLE

References:

[1] Smith, Hal L. An introduction to delay differential equations with applications to the *life sciences*. New York: Springer, 2011.

[2] Kuang, Yang, ed. *Delay differential equations: with applications in population dynamics.* Academic Press, 1993.

[3] Diekmann, O., Van Gils, S.A., Lunel, S.M. and Walther, H.O., 2012. *Delay equations: functional-, complex-, and nonlinear analysis* (Vol. 110). Springer Science & Business Media.

[4] Ruan, S. and Wei, J., 2003. On the zeros of transcendental functions with applications to stability of delay differential equations with two delays. Dynamics of Continuous Discrete and Impulsive Systems Series A, 10, pp.863-874.

[5] Toaha, S. and Hassan, M.A., 2008. *Stability analysis of predator-prey population model with time delay and constant rate of harvesting.* Punjab University Journal of Mathematics, 40, pp.37-48.

[6] Collera, J.A., 2014. *Bifurcations in delayed Lotka-Volterra intraguild predation model.* Matimyás Matematika, 37, 11–22.

[7] Shigui, R., Junjie, W. and Dongmei, X., 2017. On the distribution of zeros of a thirddegree exponential polynomial with applications to delayed biological systems. Nanjing Xinxi Gongcheng Daxue Xuebao, 9(4).

[8] Collera, J.A. and Magpantay, F.M.G., (to appear). *Dynamics of a stage structured intraguild predation model.* In Recent Advances in Mathematical and Statistical Methods for Scientific and Engineering Applications. Springer, Cham.

[9] https://www.radford.edu/~thompson/webddes/

Main Course C2: Recent Research in Dynamical System and Related Topics

Prof. Dr. Mohd Salmi Md. Noorani

Abstract: Here, I shall be talking about three distinct topics in dynamical systems. The first topic is about the growth of closed (periodic) orbit for discrete dynamical systems. This shall be done in the first two sections. The second topic, which I shall discuss in the third section, is about synchronizations of chaotic and hyperchaotic systems. And finally, in the fourth section, I shall focus on the relative new idea of persistent homology with special emphasis on its application to the analysis of time series data. The first two topics are somewhat theoretical in nature and the last one is very software dependent.

Outline:

1. Closed Orbit Counting for Discrete Dynamical Systems 1

In this lecture, I shall start with number theory by recalling the classical prime number counting theorems such as the Prime Number Theorem, Mertens Theorem and Chebotarev Theorem. I shall then focus on obtaining analogs of these number theoretic theorems for closed orbits of the well-studied dynamical system known as the shifts of finite types. I will end this talk by providing a survey of results for other related dynamical systems.

2. Closed Orbit Counting for Discrete Dynamical Systems

In this session, I shall focus on recent results with respect to newly studied dynamical systems. Other types of counting procedures are also discussed. Finally, some open problems shall be laid out to those who are interested to pursue this line of research further.

3. Synchronizations of Chaotic Dynamical Systems

In this talk, I shall provide a survey of the various synchronization schemes that can be imposed on a collection of chaotic oscillators. With these schemes come also the relevant control strategies to attain the synchronous behavior. Our main focus is to investigate sufficient conditions for synchrony to happen.

4. Topological Analysis of Dynamical Systems via Persistent Homology

Persistent homology is a tool to study topological features of point cloud data. Here our data is a time series and using Taken's embedding theorem, one can construct the so-called attractor of the dynamical system generating the time series. We investigate what information can persistent homology provide us when dealing with real-world data such as air quality, financial and stream flow data.

References:

[1] F Alsharari, MSM Noorani, H Akhadkulov. Analogues of the Prime Number Theorem and Mertens' Theorem for Closed Orbits of the Motzkin Shift. Bulletin of the Malaysian Mathematical Sciences Society, 2017, 40 (1), 307-319

[2] Ghada Al-mahbashi, MSM Noorani, Sakhinah Abu Bakar, Shahed Vahedi. Adaptive projective lag synchronization of uncertain complex dynamical networks with Disturbance. Neurocomputing, Volume 207, 26 September 2016, Pages 645-652

[3] A. Pikovsky, M. Rosenblum and M. Kurths, Synchronization: A Universal Concept in Nonlinear Sciences, Cambridge University Press, 2001.

[4] E. Munch. A user's guide to topological data analysis. Journal of Learning Analytics, 2017, 4(2).

Main Course C3: Coupling Different Modeling Approaches in the Study of Competition Ecosystems.

Dr. Nguyen Ngoc Doanh

Abstract: This lecture focuses on coupling some existing modelling approaches, such as Equation-Based Modelling (EBM), Individual-Based Modelling (IBM) and Graph-Based Modelling (GBM), in order to investigate competition ecosystems which is the most popular systems in ecology. Each modelling approach has its own strengths and weaknesses. Coupling them allows us to promote their advantages and use them in effective ways. The coupling is therefore will be able to provide deep understandings about dynamical behaviours of reference systems.

Outline:

- 1. Introduction
 - 1.1. Different Modelling Approaches
 - 1.2. A Simple Example in Ecosystems
 - 1.3. A Theoretical Case Study: A Competition Ecosystem
- 2. Equation-Based Modelling Approach
 - 2.1. Model for the Case Study
 - 2.2. Model Analysis
 - 2.3. Discussion
- 3. Individual-Based Modelling Approach
 - 3.1. Model for the Case Study
 - 3.2. Simulation Experiment
 - 3.3. Discussion
- 4. Coupling the Modelling Approaches
 - 4.1. Top-down
 - 4.2. Bottom-up
 - 4.3. A Disk Graph-Based Model
- 4.4. Discussion

Simulation Platforms: MATLAB and GAMA

(http://gama.ifi.refer.org/mediawiki/index.php/GAMA)

References:

[1] J.D. Murray, Mathematical Biology, Springer, 2002.

[2] V. Grimm and S.F. Railsback, Individual-based Modeling and Ecology. Princeton University Press, 2005.

[3] B.P. Zeigler, H. Praehofer and T.G. Kim, Theory of Modeling and Simulation, Academic Press, 2000.

[4] A.M. Law and W.D. Kelton, Simulation Modeling and Analysis, Mac Graw Hill, 2000.

[5] Doanh Nguyen-Ngoc, Tri Nguyen-Huu, Pierre Auger. Effects of Refuges and Density Dependent Dispersal on Interspecific Competition Dynamcis. International Journal of Bifurcation and Chaos, 1250029, 22(2), 2012.

[6] Doanh Nguyen-Ngoc, Rafael Bravo de la Parra, Miguel Angel Zavala, Pierre Auger. Competition and Species Coexistence in a Meta-Population Model: Can Fast Asymmetric Migration Reverse the Outcome of Competition in a Homogeneous Environment? Journal of Theoretical Biology, 256-263, 2010.

[7] Doanh Nguyen-Ngoc, Patrick Taillandier, Alexis Drogoul, Pierre Auger. Infering Equation-Based Models From Individual-Based Models. Proceeding in PRIMA Conference, India, 183-190, 2010.

[8] Doanh Nguyen-Ngoc, Thi Ha Duong Phan, Thi Ngoc Anh Nguyen, Alexis Drogoul, Jean-Daniel Zucker. Disk Graph Based-Model for Competition Dynamics. Proceeding in RIVF Conference, Vietnam, 254-257, 2010.

[9] Doanh Nguyen-Ngoc, Alexis Drogoul, Pierre Auger. Methodological Steps and Issues When Deriving Individual-Based Models from Equation-Based Models: A Case Study in Population Dynamics. Proceeding in PRIMA, Vietnam, 295-306, 2008.

Main Course C4: Dynamical Systems Analysis for Fractional Differential Equations

Assoc. Prof. Dr. Doan Thai Son

Abstract: This course introduces some fundamental aspects of the qualitative theory of fractional differential equations including the existence and uniqueness of solutions, the Lyapunov spectrum, the linearized asymptotic stability/instability theory and the invariant manifold theory.

Course Outline:

- 1. An Introduction to Fractional Differential Equations
 - 1.1. Fractional Calculus
 - 1.2. Fractional Differential Equation
 - 1.3. Existence and Uniqueness of Solutions for Fractional Differential

Equations

- 2. Asymptotic Behavior of Linear Fractional Differential Equations
 - 2.1. Mittag-Leffler Functions
 - 2.2. Linear Fractional Differential Equations
 - 2.3. Lyapunov Spectrum
- 3. Linearized Stability Theorem for Fractional Differential Equations
 - 3.1. Asymptotic Expansion of Mittag-Leffler Function
 - 3.2. Linearized Stability of Fractional Differential Equations
 - 3.3. An Instability Theorem for Fractional Differential Equations
- 4. Invariant Manifold Theorem for Fractional Differential Equations

- 4.1. Invariant Manifolds for Classical Dynamical Systems
- 4.2. Lyapunov-Perron Operator for Fractional Differential Equations
- 4.3. A Stable Manifold Theorem for Fractional Differential Equations

References:

[1] K. Diethelm; *The Analysis of Fractional Differential Equations. An Application– Oriented Exposition Using Differential Operators of Caputo Type.* Lecture Notes in Mathematics, 2004. Springer-Verlag, Berlin, 2010.

[2] Nguyen Dinh Cong, Doan Thai Son, Stefan Siegmund, Hoang The Tuan, *On stable manifolds for planar fractional differential equations, Applied Mathematics and Computation*, 226 (2014), 1, 157-168.

[3] Nguyen Dinh Cong, Doan Thai Son, Hoang The Tuan, *On fractional Lyapunov exponent for solutions of linear fractional differential equations, Fractional Calculus and Applied Analysis*, 17 (2014), 285-306.

[4] Nguyen Dinh Cong, Doan Thai Son, Siegmund Stefan, Hoang The Tuan, *Linearized asymptotic stability for fractional differential equations, Electronic Journal of Qualitative Theory of Differential Equations*, 39 (2016), 1-13.

[5] Nguyen Dinh Cong, Doan Thai Son, S. Siegmund, Hoang The Tuan, *On stable manifolds for fractional differential equations in high-dimensional spaces, Nonlinear Dynamics*, 86 (2016), 1885–1894.

[6] Nguyen Dinh Cong, Doan Thai Son, Stefan Siegmund, Hoang The Tuan, *An instability theorem for nonlinear fractional differential systems*. *Discrete and Continuous Dynamical Systems* - Series B, 22 (2017), 3079 - 3090.

Main Course C5: Dynamics and Bifurcations in Lotka-Volterra Type Systems

Dr. Kie Van Ivanky Saputra

Abstract: Interaction between populations and interactions of the populations with the environment are typically highly nonlinear. Thus, mathematical models become important tools to establish the factors underlying the temporal changes in the abundance of natural population. This short course is intended to give an introduction to the mathematical analysis of Lotka-Volterra population models. We will begin by studying in detail some examples of two species models, before moving on to general population models for the interactions of *n* species. In the latter part, we will study systems of differential equations of \mathbb{R}^n of the form

$$\dot{x}_i = x_i f(x) + e_i, \quad i = 1, 2, 3, \dots, n,$$

where e_i serves as a constant term. We will employ bifurcation analysis to analyse such system.

Course Outline:

- 1. The Growth of Single Population
 - 1.1. Malthus Model
 - 1.2. Exogenous and Endogenous Variability
 - 1.3. Other Related Models
 - 1.4. Fixed Points and Bifurcations
- 2. Predator-prey Models
 - 2.1. Lotka Volterra Model
 - 2.2. Other Type of Predator-Prey Model
 - 2.3. Kolmogorov Framework
 - 2.4. Hopf Bifurcation
- 3. Multispecies Models
 - 3.1. Competition Models
 - 3.2. Multispecies Models
- 4. Lotka Volterra Model with Constant Terms
 - 4.1. Codim-1 Bifurcations
 - 4.2. Codim-2 Bifurcations

Simulation Platforms: MATLAB, Maple and Auto2007

References:

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Talk T1: Fish Population Dynamics with Harvesting and Toxicant Effects

Dr. Hamizah Mohd Safuan

Abstract: This talk discusses fish population dynamics when subjected to harvesting and toxicant effects. Poor harvesting strategies such as overfishing/overharvesting lead to extinction of certain species of fish from the environment. Besides overfishing, toxic substances also contribute to the harmful effects on the fish population. This study considered fishery models that involve population interactions such as predation and competition with harvesting and toxicant terms. The equilibria of the model were obtained and the simulations of the bifurcation diagrams were performed to investigate the dynamical behaviours of the fish population.

Outline:

- 1. Introduction
 - 1.1. Population models
 - 1.2. Stability analysis
 - 1.3. Bifurcation analysis
- 2. Variable carrying capacity
 - 2.1. Single species models
 - 2.2. Multispecies models
 - 2.3. Analysis and results
- 3. Harvesting and toxicant effects
 - 3.1. Single species models
 - 3.2. Multispecies models
 - 3.3. Analysis and results
- 4. Optimal harvesting policy
 - 4.1. Simple case
 - 4.2. Full case
 - 4.3. Discussion

Simulation Platforms: XPPAUT, MAPLE, MATLAB

References:

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Talk T2: Mathematical Modelling and Stability Analysis of Population Dynamics

Dr. Auni Aslah Mat Daud

Abstract: This study is intended to serve as an introduction to the formulation, analysis and application of mathematical models that describe the population dynamics. In We will provide brief discussion on the definition of important terminologies and concepts in the mathematical modelling and stability analysis of the population dynamics, the aims and significance of the study and the methodologies employed in the research. Several current, existing real-world applications will be presented. A simple example of such application will be discussed in detail as a case study. The computation and numerical simulation using MATLAB will be used in the modelling and stability analysis.

Outline:

1. Introduction to Mathematical Modelling of Population Dynamics

- 1.1. What: The definition of important terminologies and concepts
- 1.2. Why: The aims of the study
- 1.3. How: The methodologies employed
- 2. Introduction on Stability Analysis of Population Dynamics
 - 2.1. What is stability analysis?
 - 2.2. Why do we need stability analysis?
 - 2.3. How to perform local stability analysis?

3. Simple Compartmental Models: Real World Applications

3.1. The formulation of the mathematical models: Linear and nonlinear models

3.2. The local stability analysis

- 4. Several Common Mistakes
 - 4.1. Mathematically well-posed
 - 4.2. The physical meaning of the model
 - 4.3. Reasonable model parameter values

Simulation Platforms: MATLAB

References:

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[2] Garnett, G. P. 2002. An introduction to mathematical models in sexually transmitted disease epidemiology. Sexually transmitted Infection-BMJ Journals. 78(1):7-12.

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Talk T3: Stability Index for the Characterization of Riddled Basin in a Coupled Dynamical System

Dr. Ummu Atiqah Mohd Roslan

Abstract: We consider a coupled dynamical system with a Milnor attractor whose basin of attraction is riddled with the basin of a second attractor. We first study how the global geometry of the basin of attraction changes as we vary the parameter in the system. Secondly, we focus on the local geometry of the riddled basin of attraction. To characterize this riddled basin, we compute a stability index for the attractor in the system. Our numerical results show that for Lebesgue almost all points in the attractor, the index is positive for some parameter region where the riddled basin occurs.

Outline:

- 1. Definitions of stability
 - 1.1. Invariant set
 - 1.2. Lyapunov stable
 - 1.3. Asymptotically stable
- 2. Basin of attraction

- 2.1. Definition
- 2.2. Riddled and intermingled basins
- 3. What is attractor?
 - 3.1. Weak attractor
 - 3.2. Milnor attractor
 - 3.3. Asymptotically stable attractor
- 4. Example of dynamical system with riddled basin
 - 4.1. Ashwin's model
- 5. What is stability index?
 - 5.1. Definition
 - 5.2. Results on stability index for Ashwin's model
 - 5.3. Discussion

Simulation Platforms: MATLAB

References:

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for a Riddled Basin Attractor of a Piecewise Linear Map, Dynamical Systems: An International Journal, DOI:10.1080/14689367.2016.1148662, 2016

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Talk T4: Dynamical Systems Analysis of Local and Non-Local Dispersal Models

Dr. Mohd Hafiz Mohd

Abstract: In this talk, we discuss the effects of different dispersal patterns on the occurrence of priority effects (alternative stable states) and coexistence in multi-species communities by employing local (partial-differential equations) and non-local dispersal (integro-differential equations) models. Our analysis shows the existence of a threshold value in dispersal strength (i.e. saddle-node bifurcation) above which priority effects disappear. These results also reveal a co-dimension 2 point, corresponding to a degenerate transcritical bifurcation: at this point, the transcritical bifurcation changes from subcritical to supercritical with corresponding creation of a saddle-node bifurcation curve.

Outline:

- 1. Introduction
 - 1.1. Problem Description and Ecological Background
 - 1.2. Experimental Observation I
 - 1.3. Experimental Observation II
- 2. Overview of Selected Mathematical Models
 - 2.1. Modelling Biotic Interactions Between Species
 - 2.2. Modelling Dispersal Process: Local vs. Non-Local Dispersal Systems
 - 2.3. Modelling Background of the Study
- 3. Biotic Interactions and Abiotic Environments
 - 3.1. A Deterministic Model with Biotic Interactions and Environmental Suitability
 - 3.2. Analytical Results on the Range Limits of Species
 - 3.3. Numerical Results
- 4. Local and Non-Local Dispersal Models and Outcomes of Species Interactions
 - 4.1. A Deterministic Model with Biotic Interactions and Local Dispersal
 - 4.2. A Deterministic Model with Biotic Interactions and Non-Local Dispersal
 - 4.3. Comparative Studies of Local and Non-Local Dispersal Models

Simulation Platforms: XPPAUT, MAPLE, MATLAB

References:

[1] Mohd, M. H., Murray, R., Plank, M. J., & Godsoe, W. (2018). Effects of different dispersal patterns on the presence-absence of multiple species. Communications in Nonlinear Science and Numerical Simulation, 56, 115-130.

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Hands-on Workshop: Computational Dynamical Systems Using XPPAUT & AUTO

Dr. Kie Van Ivanky Saputra & Dr. Mohd Hafiz Mohd

Abstract: In these hands-on workshops, we will discuss some techniques in numerical continuation and bifurcation analysis using XPPAUT and Auto. These are free packages for numerically solving and analyzing dynamical systems. The problems that can be solved using these tools can range from deterministic ordinary-differential equations to stochastic models, to partial differential and integro-differential equations. The trainers will teach the participants how to get started and beyond with XPPAUT and Auto packages. The workshop starts with a gentle introduction and how to install these software, and an overview of the numerical routines, followed by solving the steady-states, graphing and plotting. Thereafter sessions gradually increase in complexity, covering general steps in bifurcation analysis and how to compute complete bifurcation diagrams, particularly co-dimension one and co-dimension two bifurcation plots.

VII. Feedbacks and Conclusion

Below are some of the surveys and feedbacks from our attendees:



In term of participants' background, the school hosted nearly 70% postgraduate students, 17% lecturers and 13% senior undergraduate students. The gender balance in terms of participants of the school was achieved with 50% of participants were female. Most of the attendees got to know about the school through the promotion emails and also the word of mouth.



Based on the comments received via our online feedback portal, the attendees felt that the programme was a great success. The school ran smoothly and achieved its learning objectives / outcomes, and the participants enjoyed the great courses, interesting talks together with hands-on workshop sessions. They had a very good atmosphere for discussion and got various opportunities to network with each other. In the main, they also learnt the importance of the theoretical and numerical aspects of dynamical systems and bifurcation analysis in analysing their mathematical models, plus their applications in different fields such as biology, physics and engineering. Here are some of the responses about their personal expectations when they decided to join the school and whether these expectations have been met by the SEAMS School 2018 on DySBA:

ID↑	Name	Responses
1	anonymous	Talks are diverse and great.
2	anonymous	The topics are well-thought. Each topic jives effectively with the other topics. I learned so much from the lectures and talks. So far, the best SEAMS school I attended in terms of the organization of the lectures.
3	anonymous	Hoping to have more time to learn about xppaut
4	anonymous	One of workshop cannot be done due to installation of software are having dificulties
5	anonymous	I have known a lot of theoretical and practical aspects of bifurcation thanks to the workshop.
6	anonymous	The material & program is very good. Maybe the length duration too long (monday untill saturday 8am - 6pm) so i think most of participants feel tired
7	anonymous	I am happy that nearly all my expectations were met.
8	anonymous	I learn a lot and some talks related to my research and other talks also contribute in giving knowledge about other area of research that can be explored in future.
9	anonymous	For Auto workshop, it would be better if we can run it in the lab instead of listening the introduction of AUTO.
10	anonymous	To meet researchers working on my field of interest
11	anonymous	1- Can I joining next time? 2-How can I apply for Post-Doctor research program at USM? 3- I love to research so much.
12	anonymous	Some new knowledge were shared need time to digest.

Overall, their comments are constructive and most of them gave positive responses to the school. One of the weaknesses of the school was about the lab demonstration on Auto software. Due to the incompatibility issue and the short timing of the workshop, we had some technical problems in installation of Auto. So, the computer lab cannot be conducted via hands-on mode; however, the lab instructor i.e. Dr KVI Saputra tried his best during the lab demonstration session to illustrate the important features of Auto using his laptop. He also provided some of the notes and lab manual to participants, so that if they were keen to employ this software in their research, they could try it on their own. Luckily, another hands-on workshop conducted by Dr Mohd Hafiz Mohd on the use of XPPAUT in conducting bifurcation analysis ran according to the plan. The XPPAUT workshop started with a gentle introduction on how to install the software, and later the participants were guided on the numerical routines and also how to write the coding in order to solve differential equations. This session was followed by solving the steady-states, graphing and plotting. The attendees were also exposed to the general steps in bifurcation analysis and how to compute complete bifurcation diagrams, particularly co-dimension one and co-dimension two bifurcation plots. We thank all attendees for their constructive and detailed feedbacks, and we will surely improve ourselves in the future events.

In summation, this school has become a platform for knowledge sharing session, apart from providing numerous opportunities for participants and speakers to communicate and network in a collaborative environment. We would like to thank USM, PPSM, committee members, attendees, speakers and our sponsors for helping to make this SEAMS School 2018 on DySBA a success. Thank you.

IX. Photos

Courses/Lectures of SEAMS School 2018 on DySBA



First Course of the SEAMS School 2018 on DySBA: Introduction to Delay Differential Equations with Applications by Assoc. Prof. Dr. Juancho Collera.





Interesting talk by Prof Mohd Salmi from UKM on the Synchronisation of Chaotic Systems and Persistent Homology.



Day-3 Course for the SEAMS School 2018 on DySBA by Dr Nguyen Ngoc Doanh. The course was on the dynamical behaviours of agent-based modelling and equation-based modelling.



Day-4 Course for the SEAMS School 2018 on DySBA: Dynamical Systems Analysis of Fractional-Order Models by Assoc. Prof. Dr Doanh Thai Son from Institute of Mathematics, Vietnam Academy of Science and Technology.



Day-5 Course for the SEAMS School 2018 on DySBA: Dynamics and Bifurcations in Lotka-Volterra Type Systems by Dr. Kie Van Ivanky Saputra from Universitas Pelita Harapan, Indonesia. We really enjoyed the course by Dr Ivanky, which emphasises on the state-of-the-art in dynamical systems and bifurcation analysis.

Contributed Talks of SEAMS School 2018 on DySBA



SEAMS School 2018 on DySBA contributed talk sessions on dynamical systems and bifurcation analysis by our invited speakers: Dr. Hamizah Mohd Safuan (UTHM), Dr. Auni Aslah Mat Daud & Dr. Ummu Atiqah Mohd Roslan (UMT) and Dr. Mohd Hafiz Mohd (USM)

Quiz Sessions of SEAMS School 2018 on DySBA



SEAMS DySBA Quiz sessions. Some of the winners were Aditya Firman and Livia Owen both from Institut Teknologi Bandung, Indonesia. Well done guys! The prizes were sponsored by The TOP Penang.

Participant's Talks Competition during SEAMS School 2018 on DySBA



Some of the presentation during participant's talks competition. The participants that entered this competition were Jay Michael R. Macalalag (Philippines), Meach Mon (Cambodia), Mahmoud Ahmed Mohamed Abdelaziz (USM, Malaysia), Elshahed Mahmoud Moustafa (USM, Malaysia) and Phan Thanh Hong (Vietnam).





The Best Presenters were awarded to Jay Michael R. Macalalag (Philippines) and Elshahed Mahmoud Moustafa (USM, Malaysia).

SEAMS School 2018 on DySBA Grand Dinner



The participants, speakers and committee members enjoyed great food during SEAMS School 2018 on DySBA Grand Dinner, plus with fun activities.

SEAMS School 2018 on DySBA Penang Excursion Programme





The participants, speakers and committee members enjoyed themselves during excursion programmes by visiting Georgetown, one of UNESCO World Heritage sites.



Closing Ceremony of the SEAMS School 2018 on DySBA

The end of the SEAMS School 2018 on DySBA. Your presence helps to make this event a success and your enthusiasm and positive spirit help to make our time together both productive and fun. We wish you all the best in your future undertakings!

SEAMS School 2018 on DySBA Collaborators and Sponsors

South East Asian Mathematical Society



Centre International De Mathématiques Pures Et Appliquées



International Mathematical Union



Division of Research and Innovation, Universiti Sains Malaysia



Department of Higher Education, Ministry of Education, Malaysia



Malaysian Mathematical Sciences Society



Penerbit Universiti Sains Malaysia



Penang State Museum



Penang Convention & Exhibition Bureau



Ride Safe with MULA



Interactive Presentation Software



The Kapit's Restaurant



This report is prepared by:

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