Scientific Report of
CIMPA School on Finsler Geometry and Applications

Title of the school : CIMPA School on Finsler Geometry and Applications

Dates : 05-15 December, 2019

Location : DST- Centre for Interdisciplinary Mathematical Sciences, Institute of Science, BHU Varanasi-221005, INDIA.

Organizer: Bankteshwar Tiwari, DST- Centre for Interdisciplinary Mathematical Sciences, Institute of Science, BHU Varanasi-221005, INDIA.

Co-Organizer : Athanase Papadopoulos, Institut de Recherche Mathematique Avancee, Universite de Strasbourg et CNRS, France.

Members of scientific Committee:

- Athanase Papadopoulos (Institut de Recherche Mathematique Avancee, Universite de Strasbourg et CNRS, France)
- Norbert A'Campo (Univ. Basel, Switzerland)
- S.G. Dani (UM-DAE Center for Excellence in Basic Sciences, Mumbai, INDIA)
- Krishnendu Gangopadhyay(IISER, Mohali, INDIA)
- Riddhi Shah (JNU, New Delhi, INDIA)

I. Summary

The CIMPA School on "**Finsler Geometry and Applications**" was organized by DST-Centre for Interdisciplinary Mathematical Sciences, Institute of Science, BHU organized a during Dec. 05-15, 2019. There were seven coordinated courses related to current research on Finsler Geometry. These courses were taught by renowned experts of the field, where as two supplementary courses, first was delivered by Dr. Soma Maiti (IISER, Mohali) to compliment the basic need of the some advanced courses and second by Dr. Alena Zhukova (Russia) on Moduli Spaces of Planar Linkages. There was also a research presentation session by participants in which three participants Dr. Ebtsam Hassan Taha Mohamed (Egypt), Ms Samaneh Saberali(IRAN) and Dr. Louis Merlin(Germany) presented their research work. There were 55 registered participants including 5 CIMPA participants and 26 participants from various outstation institutions of the country like TIFR Mumbai, IMSc Chennai, CMI Chennai, IISER Mohali, IISER Bhopal, IISER Tirupati, Central University of Punjab etc. Remaining 24

participants were from different institutions of Varanasi. Out of 55 participants 16 were female whereas 39 were male participants. There were ten working day. Each day five, one hour, lectures have been delivered. On 10th December there was a CIMPA trip, of almost all speakers and participants, to Sarnath (a tourist place about 20 km from Varanasi). On last day there was a Valedictory session in which most of the speakers and participants were of the opinion that they enjoyed academically as well as their stay during the whole school. In short the school was successful and achieved its goal. We are planning to follow up this school by offering CIMPA courses this year by two experts Prof. Athanase Papadopoulos and Prof.Norbert A'Campo who were already speakers in the above CIMPA School.

II. Scientific content

	Speaker	Courses
1	Prof. Athanase Papadopoulos (Institut de Recherche Mathematique Avancee, Universite de Strasbourg et CNRS, France)	Metric Finsler geometry **(5 Lectures)**
2	Prof.Norbert A'Campo (Univ. Basel, Switzerland)	Riemann Surfaces: An introduction **(5 Lectures)**
3	Prof.Gilles COURTOIS (University of Paris, France)	Compactness and finiteness results for Gromov hyperbolic spaces **(5 Lectures)**
4	Prof. Gérard BESSON (Universite de Grenoble, France)	Compactness and finiteness results for Gromov hyperbolic spaces **(5 Lectures)**
5	Prof. Ivan IZMESTIEV (Technical University of Vienna, Astria)	Mixed volumes and inequalities **(6 Lectures)**
6	Prof. Olga Kharlampovich (City University of New York, Hunter College and Graduate Center, USA)	Groups actions on $\R$-trees and Λ-trees **(5 Lectures)**
7	Prof. Muhammed Uludag(Galatasaray University, Turkey)	$PSL(2,\mathbb Z)$ **(5 Lectures)**
8	Prof. Kenichi OHSHIKA(Gakushuin University, Tokyo, Japan)	Actions of mapping class groups on various spaces and their rigidity **(5 Lectures)**
9	Dr. Alena Zhukova(Saint-Petersburg State University, Russia) [CIMPA participant]	Moduli Spaces of Planar Linkages **(2 Lectures)**
10	Dr. Soma Maiti(IISER Mohali, India)	Bishop-Gromov Volume comparison Theorem

		(4 Lectures)

Summary of the Courses:

Course 1 Metric Finsler geometry

Speaker: Athanase Papadopoulos

Summary:

This was a series of lectures on metric theory of Finsler geometry, that is, Finsler geometry withouttensors. The examples of metric spaces and Finsler metric spaces were introduced explaining fundamental resultsand presenting the basic questions. The examples include several metrics defined on convex sets, includingthe Apollonian metric, the part metric, the Funk metric and the Hilbert metric. There was no pre-requisite, except for undergraduate mathematics.

Course 2 Riemann Surfaces: An introduction

Speaker: Norbert A'Campo

Summary:

Riemann Surfaces appear at many places in science. This course was an introduction. The topics discussed were basic theorems of Riemann surfaces such as the Riemann-Roch and Uniformisation Theorem. The course was planned to be self-contained at the level of Masters in Mathematics.

Course 3 Compactness and finiteness results for Gromov hyperbolic spaces

Speakers: This course was jointly co-ordinated by two speakers Gerard Besson and Gilles Courtois.

Summary:

They discussed the δ-hyperbolic spaces, (X, d), endowed with a proper action of a subgroup of its isometry group. It was assumed that the quotient is compact with diameter bounded by D and that the entropy of (X, d), is bounded above by a real number $H > 0$. In this context they proved a version of the celebrated Bishop-Gromov inequality. It shows, in particular, that H plays the same role as a lower bound on the Ricci curvature for Riemannian manifolds and that δ the one of an upper bound on the sectional curvature. From this central inequality they deduced, with extra assumptions, compactness as well as finiteness results.

They followed mainly these two main references below:

- Coornaert-Delzant-Papadopoulos: Geometrie et Theorie des Groupes Lecture Notes in Mathematics 1441, Springer.
- Bridson- Haeiger: Metric spaces of non-positive curvature, Springer, Grundlehren der Mathematischen Wissenschaften, 319.

Course 4 Mixed volumes and inequalities

Speaker: Ivan Izmestiev (Technical University of Vienna, Astria)

Summary: Theory of mixed volumes provides a proof of the isoperimetric inequality for convex bodies in the Euclidean space of arbitrary dimension. It also creates a framework for the study of curvature measures on the boundaries of non-smooth convex bodies, such as the "total mean curvature" of a polyhedron. In this course he gave an introduction to the mixed volumes and proved the Brunn-Minkowski and the Minkowski inequalities.

Besides the necessary knowledge of linear algebra and calculus, some knowledge of differential geometry of submanifolds of the Euclidean space was required.

Course 5 Groups actions on R-trees and λ-trees

Speaker: Olga Kharlampovich

Summary: Bass-Serre theory relates group actions on trees with decomposing groups as iterated applications of the operations of amalgamated product and HNN extension, via the notion of the fundamental group of a graph of groups.

One of the generalizations of Bass-Serre theory is the theory of isometric group actions on real trees (R-trees) which are metric spaces generalizing the graph-theoretic notion of a tree. Group actions on R-trees arise naturally in geometric topology, in the study of Culler-Vogtmann's Outer space, Thurston's Hyperbolization Theorem for Haken 3-manifolds, as well as in geometric group theory. Asymptotic cones of groups often have a tree-like structure and give rise to group actions on real trees. The use of R-trees and λ-trees, in particular Z^n-trees, together with Bass-Serre theory, are key tools in the work on the elementarytheory of a free group by Kharlampovich-Miasnikov and Sela.

She has covered the following topics:

1. Actions on simplicial trees. Amalgamated products and HNN extensions, Bass-Serre theory, graphs of groups. Action of $SL_2(Z)$ on the hyperbolic plane.

2. Ordered abelian groups Λ. Actions on Λ-trees, theory of a single isometry.

3. *R*-trees. Rips' theorem: Let *G* be a finitely generated group with a free action on an *R*-tree. Then G is a free product of surface groups and free abelian groups.

4. Ideas of the proof of the theorem, Bestvina-Feighn-Rips machine.

5. Structure theorems for finitely generated groups acting freely on Z^n-trees and R^n-trees. Finitely presented groups acting freely on Λ-trees.

Course 6 Actions of mapping class groups on various spaces and their rigidity

Speaker: Kenichi OHSHIKA (Gakushuin University, Tokyo, JAPAN)

Summary:

In this course, he discussed about various metrics on Teichmuller space and its combinatorial analogues, and the mapping class group actions on them. He started with introducing the original definition of Teichmuller space, which is naturally equipped with what is called the Teichmuller metric, and present Royden's theorem on its rigidity with respect to the mapping class group action.

Course 7: *PSL(2,Z)*

Speaker: Muhammed Uludag (Galatasaray University, Turkey)

Summary:

He started from the basic definitions and explained the connection between the modular group and the continued fractions. From the action of *PSL(2,Z)* on the plane tree, he gave a concrete description of the category of its subgroups, Sub*PSL(2,Z)*and explore the panorama of coverings of the modular curve. He also discussed several actions of *PSL(2,Z)*i.e. on Sub*PSL(2,Z)*via conjugation on the upper half plane via unimodular Mobius transformations and on binary quadratic forms via change of variables.

Supplementary Lectures:

Course: Bishop-Gromov Volume comparison Theorem

Speaker: Soma Maity, IISER Mohali

Summary:

In first two lectures she introduced Riemannian manifolds, geodesics, curvature, exponential map, Jacobi fields. Then she gave a proof of volume comparison theorem. The pre-requisite of this lecture series is elementary differential geometry.

Course: Moduli Spaces of Planar Linkages

Speaker: Alena Zhukova (Russia)

Summary:

Linkages are classical mechanical objects with a lot of applications. In her talk she explained how configuration (moduli) spaces of linkages are constructed and give some examples of them. She also discussed some topological and algebraic results on these objects.

Time Table of CIMPA School on Finsler Geometry and Applications

(5-15 December 2019)

Venue: DST-Centre for Interdisciplinary Mathematical Sciences, Institute of Science, BHU Varanasi

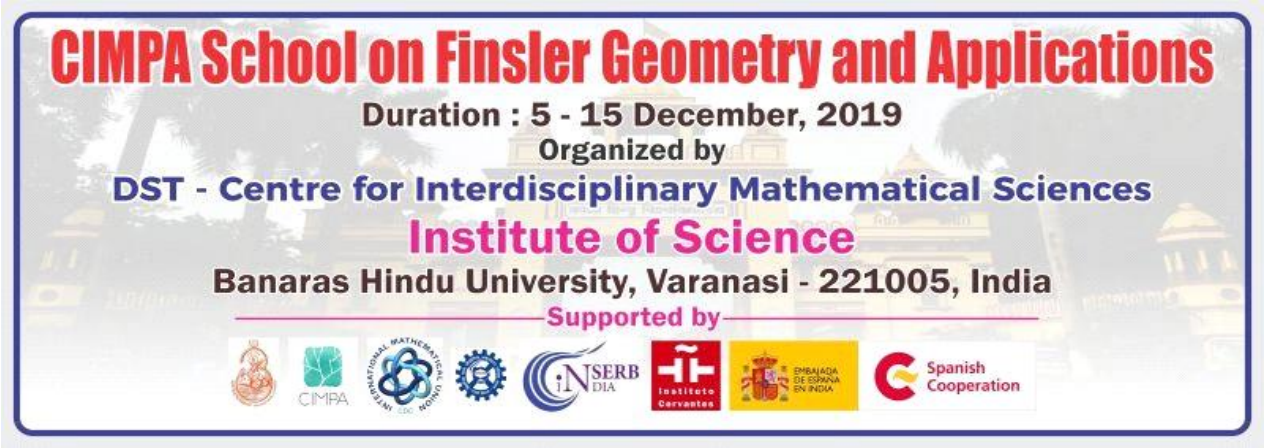
5th Dec (Thursday)	(9:00-10:30) Registration/ Inauguration/ Tea break	(10:30-11:30) Papadopoulos	(11:30-12:30) Soma	(12:30-13:30) A'Campo	(13:30-15:00) Lunch Break	(15:00-16:00) Soma	16:00-16:30 (Tea/Coffee Break)	(16:30-17:30) Papadopoulos
	9:30-10:30	**10:30-11:00 (Tea/Coffee Break)**	**11:00-12:00**	**12:00-13:00**	**Lunch Break 13:00-14:30**	**14:30-15:30**	**15:30-16:00 (Tea/Coffee Break)**	**16:00-17:30**
6th Dec (Friday)	Papadopoulos		Soma	A'Campo		Besson		IZMESTIEV
7th Dec(Saturday)	Soma		OHSHIKA	OHSHIKA		Besson		IZMESTIEV
8th Dec(Sunday)	COURTOIS		OHSHIKA	Kharlampovich		Besson		IZMESTIEV
9th Dec(Monday)	COURTOIS		OHSHIKA	Kharlampovich		Besson		IZMESTIEV
10th Dec (Tuesday)								
11th Dec(Wednesday)	COURTOIS		OHSHIKA	Kharlampovich		Besson		Uludag
12th Dec (Thursday)	COURTOIS		IZMESTIEV	IZMESTIEV		Uludag		Uludag
13th Dec(Friday)	COURTOIS		Papadopoulos	Kharlampovich		A'Campo		Uludag
14th Dec(Saturday)	Uludag		Papadopoulos	Kharlampovich		A'Campo		Zhukova
15th Dec(Sunday)	Zhukova		A'Campo	Presentation by Participants		**Valedictory**		

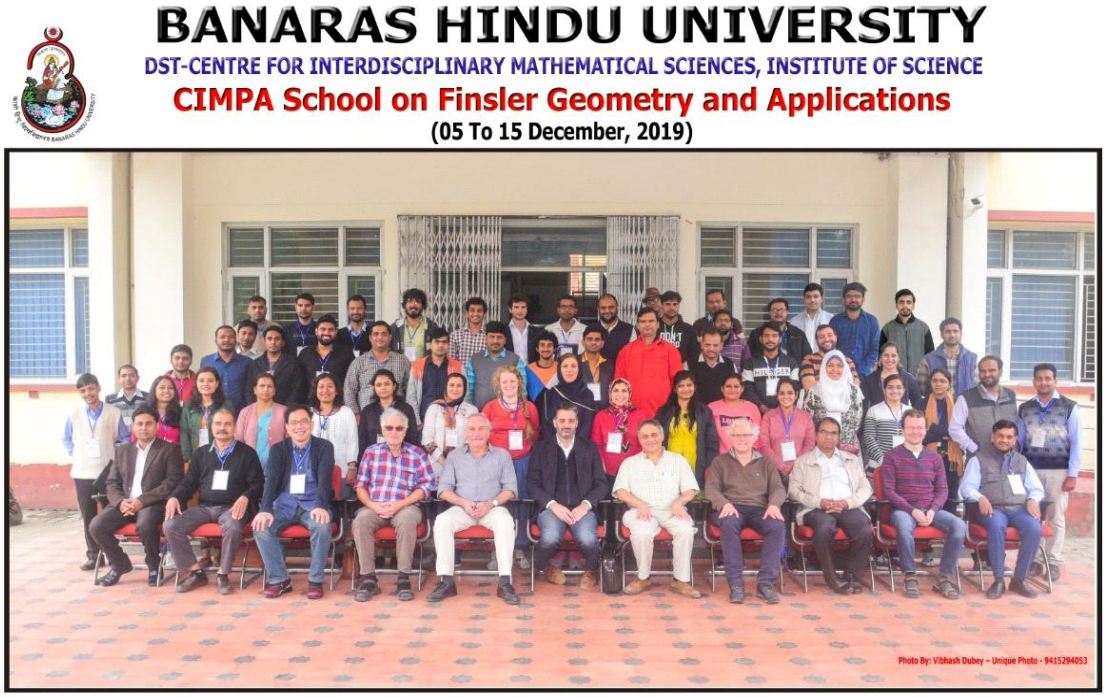
III. Participants

S.N	Participant
1	Dr. Alena Zhukova(Saint-Petersburg State University, Russia) [CIMPA participant]
2	Ms. Tayebeh Tabatabaeifar(Amirkabir Univ of Technology,Tehran,Iran) [CIMPA participant]
3	Dr. Ebtsam Hassan Taha Mohamed (Cairo University; Egypt and HRI,Allahabad) [CIMPA participant]
4	Ms. Mastooreh Farahmandy Motlagh(University of Mazandaran,IRAN) [CIMPA participant]
5	Ms Samaneh Saberali(Urmia University, Iran) [CIMPA participant]
6	Ms.Sarita Rani, Central University of Punjab, Bathinda, Punjab (INDIA)
7	Ms Ramandeep Kaur, Central University of Punjab, Bathinda, Punjab (INDIA)
8	Ms Seema, Central University of Punjab, Bathinda, Punjab (INDIA)
9	Dr. Bandana Das, Muralidhar Girls College, Kolkata, (INDIA)
10	Dr. Ananya Chaturvedi,TIFR, MUMBAI (INDIA)
11	Dr. Gauree Shanker, Central University of Punjab, Bathinda, Punjab (INDIA)
12	Dr. Ghanashyam Kumar Prajapati, LNJPIT Chhapara, Bihar (INDIA)
13	Mr. Umar MohdKhan, AMU, Aligarh(INDIA)
14	Mr. Ankit, Central University of Punjab, Bathinda, Punjab (INDIA)
15	Dr. Pradip Majhi, Univ of Kolkata (INDIA)
16	Mr. Debanjan Nandi,TIFR, MUMBAI (INDIA)
17	Mr. Suraj Krishna M. S., TIFR, MUMBAI(INDIA)
18	Mr. Gianluca Faraco, TIFR, MUMBAI(INDIA)
19	Mr. George Shaji, IISER MOHALI (INDIA)
20	Mr. Kuldeep Saha, IISER BHOPAL(INDIA)
21	Mr. Neeraj Kumar Dhanwani, IISER BHOPAL(INDIA)
22	Mr. Gautam Neelakantan M, IISER MOHALI (INDIA)
23	Mr. Shreyas Nagabhushana Samaga, IISER BHOPAL (INDIA)
24	Mr. Tejbir, IISER MOHALI (INDIA)
25	Mr. Himalaya Senapati, CMI,Chennai (INDIA)
26	Dr. Parth Sarthi Ghosh, Presidency University, Kolkata(INDIA)

27	Dr. Soumya Dey, I.M.Sc., Chennai (INDIA)
28	Dr. Kamlesh Kumar Dubey, Invertis University,Bareilly (INDIA)
29	Dr. Brijesh Kumar Tripathi, LE College Morbi, Gujarat, (INDIA)
30	Dr. H.A. Gururaja, IISER, Tirupati (INDIA)
31	Dr. Louis Merlin, Aachen University (Germany)
32	Dr. Ranadip Gangopadhyay, Varanasi (INDIA)
33	Mr. Angit S, Banaras Hindu University, Varanasi (INDIA)
34	Ms. Priya Kumari, Banaras Hindu University, Varanasi (INDIA)
35	Mr. Ashok kumar, Banaras Hindu University, Varanasi (INDIA)
36	Ms. Anjali Shriwastawa, Banaras Hindu University, Varanasi (INDIA)
37	Dr. Rakesh Raushan, Banaras Hindu University, Varanasi (INDIA)
38	Mr. Santosh Kumar, Banaras Hindu University, Varanasi (INDIA)
39	Mr. Sachin kumar, Banaras Hindu University, Varanasi (INDIA)
40	Mr. Kapish Chand Meena, Banaras Hindu University, Varanasi (INDIA)
41	Ms. Kiran Meena, Banaras Hindu University, Varanasi (INDIA)
42	Mr. Mukesh Kumar, Banaras Hindu University, Varanasi (INDIA)
43	Mr. Arpit Dwivedi, Banaras Hindu University, Varanasi (INDIA)
44	Ms. Navneet Kaur, Banaras Hindu University, Varanasi (INDIA)
45	Mr. Pankaj Kumar, Banaras Hindu University, Varanasi (INDIA)
46	Dr. Buddhdev Pal, Banaras Hindu University, Varanasi (INDIA)
47	Dr. Akhilesh Yadav, Banaras Hindu University, Varanasi (INDIA)
48	Prof. Mukut Mani Tripathi, Banaras Hindu University, Varanasi (INDIA)
49	Dr. Rakesh Ranjan, Banaras Hindu University, Varanasi (INDIA)
50	Dr. Raghavendra Chaubey, Banaras Hindu University, Varanasi (INDIA)
51	Dr. Ganga Ram, Banaras Hindu University, Varanasi (INDIA)
52	Ms. Shiwani Singh, Banaras Hindu University, Varanasi (INDIA)
53	Dr. M S Panwar, Banaras Hindu University, Varanasi (INDIA)
54	Dr. Sangeeta Srivastava, HC PG College Varanasi(INDIA)
55	Dr. Tadkeshwar Nath Mishra, Banaras Hindu University, Varanasi (INDIA)

IV. Pictures





Sitting Row (L to R) : Dr. Raghavendra Chaubey, Prof. Mukut Mani Tripathi, Prof, Ken'ichi Ohshika, Prof. Norbert A' Campo, Prof Gérard Besson, Prof. Ludovic Rifford, Prof. Athanase Papadopoulos, Prof. Gilles Courtois Prof. S. K. Upadhyay, Prof. Ivan Izmestiev, Prof. Bankteshwar Tiwari

Standing Row 1st : Dr. T.N. Mishra, Ms. Anjali Shriwastawa, Ms. Shivani Singh, Dr. Sangita Srivastava, Dr. Soma Maity, Dr. Bandana Das, Ms. Tayebeh Tabatabaeifar, Dr. Alena Zhukova, Ms. Mastooreh Farahmandy Motlagh, Ms. Samaneh Saberali Ms. Sarita Rani, Ms. Ramandeep Kaur, Ms. Seema, Dr. Ebtsam Hassan Taha Mohamed, Ms. Navneet Kaur, Ms. Kiran Meena, Dr. Rakesh Ranjan, Dr. Ghanshyam Prajapati

Standing Row 2nd : Dr. Ganga Ram, Dr. Ranadip Gangopadhyay, Dr. Rakesh Raushan, Mr. Angit S, Mr. Mukesh Kumar, Dr. Pradip Majhi, Dr. Buddhadev Pal, Dr. Akhilesh Yadav, Shreyas Nagabhushana Samaga, Dr. Brijesh Kumar Tripathi Dr. Kamlesh Kumar Dubey, Dr. Gauree Shanker, Dr. Ankit Yadav, Dr. Gianluca Faraco, Dr. Ananya Chaturvedi, Dr. M. S. Panwar

Standing Row 3rd : Mr. Ashwani Kumar Tiwari, Mr. Kapish Chand Meena, Mr. Santosh Kumar, Mr. George Shaji, Mr. Gautam Neelakantan, Dr. Louis Merlin, Mr. Himalya Senapati, Mr. Umar Mohd Khan, Mr. Pankaj Kumar, Mr. Sachin Kumar Mr. Partha Sarathi Ghosh, Mr. Ashok Kumar, Mr. Tejbir, Mr. Arpit Dwivedi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B. Tiwari
Ludovic Rifford
K. Shukla


Results of Dahmani and Groves imply the following two corollaries.
Corollary
Let G be a finitely presented regular Λ-free group. Then:
G has a non-trivial abelian splitting and one can find such a splitting effectively,
G has a non-trivial abelian JSJ-decomposition and one can find such a decom
ely.
Corollary
The isomorphis
ge class of finitely presented groups
ely on some Λ-tree.
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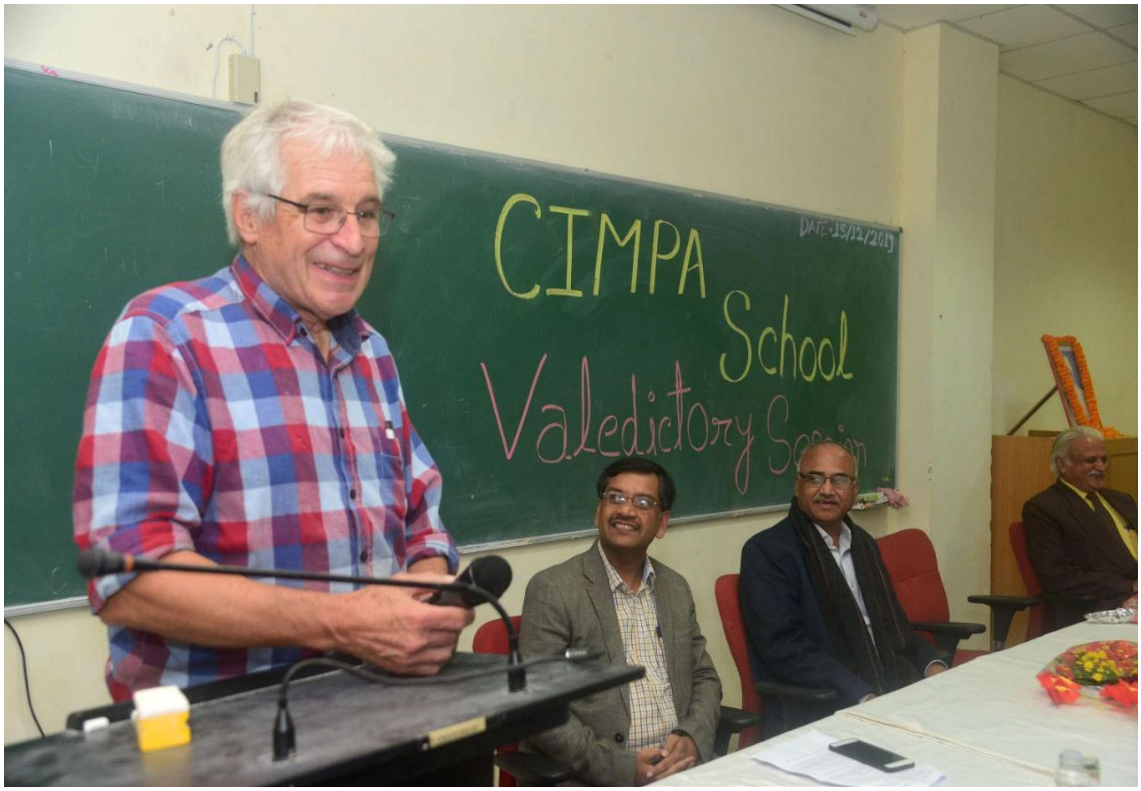
CIMPA
School
Valedictory Session
DATE: 19/12/2019