# Vietnam Academy of Science and Technology Institute of Mathematics

# HYPERPLANE ARRANGEMENTS: RECENT ADVANCES AND OPEN PROBLEMS March 11 - March 22, 2019 CIMPA-IMH reaseach school

Program book

Hanoi, 2019

# HYPERPLANE ARRANGEMENTS: RECENT ADVANCES AND OPEN PROBLEMS CIMPA-IMH reaseach school

The theory of hyperplane arrangements is a very active area of research. In the recent years there has been huge progress in the understanding of this subject. Some specific

- 1. Are the monodromy operators or at least the Betti numbers  $b_m(F)$  of the Milnor fiber F of a hyperplane arrangement  $\mathcal{A}$  are combinatorially determined?
- 2. Are the jump loci of the cohomology of the complement of  $\mathcal{A}$  with coefficients in rank one local systems determined by the intersection lattice  $L(\mathcal{A})$ ?
- 3. Can there be torsion in the first homology group?

main problems are

- 4. Terao's Conjecture: Is the freeness of an arrangement combinatorially determined?
- 5. Matroids and Kazhdan-Lusztig polynomials: Are the coefficients of the Kazhdan-Lusztig polynomials associated to non-realizable matroids always non-negative?

The aim of this CIMPA school is to introduce this fascinating area of mathematics to researchers from Viet Nam and neighboring countries in Asia. The above problems are among various aspects of Hyperplane Arrangements will be introduced at this CIMPA school through 6 carefully selected courses. The school is primarily oriented towards PhD students and young researchers working in the area of Algebra, Geometry and Topology.

The school is followed by a 2 days workshop, where most recent progresses in the field will be presented by experts from all over the world. The schools participants are also encouraged to participate in the workshop. They can take this opportunity to exchange ideas with experts, as well as to approach some concrete and open problems which can be the subject of their research in the future.

#### Organizers

The scientific coordinators of the school:

- Alexandru Dimca, University Noice Sophia Antipolis, Nice, France
- Nguyen Viet Dung, Institute of Mathematics, VAST, Hanoi, Vietnam

The scientific committee of the school:

- Alexandru Suciu, Northeastern University, Boston, USA Chair
- Anca Daniela Macinic, Institute of Mathematics, Romanian Academy, Bucharest, Romania
- Ngô Viêt Trung, Institute of Mathematics, VAST, Hanoi, Vietnam

The local organizers are:

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#### Sponsor

- 1. Centre International de Mathématiques Pures et Appliquées (CIMPA)
- 2. Commission for Developing Countries International Mathematical Union (CDC-IMU)
- 3. Institute of Mathematics, Hanoi (IMH)
- 4. Vietnam Academy of Science and Technology (VAST)
- 5. International Centre for Research and Post graduate Training in Mathematics under the auspices of UNESCO (ICRTM)

# SCHOOL PROGRAM

### Monday, March 11, 2019

## Morning

08:30 - 08:45	Registration
08:45 - 09:00	Opening Ceremony
09:00 - 10:50	<b>Daniel Cohen</b> (Louisiana State University, USA) On arrangement groups and associated invariants Lecture 1 and 2
10:50 - 11:10	Coffee break
11:10 - 12:00	<b>Graham Denham</b> (University of Western Ontario, Canada) Arrangements, wonderful models and toric varieties Lecture 1
12:00 - 14:00	Lunch
A fternoon	
14:00 - 14:50	Clément Dupont (University of Montpellier, France) On the monodromy of Milnor fibers of hyperplane arrangements Lecture 1
14:50 - 15:10	Coffee break
15:10 - 16:00	Clément Dupont (University of Montpellier, France) On the monodromy of Milnor fibers of hyperplane arrangements. Lecture 2

16:00 - 16:45 **Tutorial session** 

## Tuesday, March 12, 2019

## Morning

09:00 - 09:50	<b>Graham Denham</b> (University of Western Ontario, Canada) Arrangements, wonderful models and toric varieties Lecture 2
10:00 - 10:50	Max Wakefield (United States Naval Academy, USA) Matroids, arrangements, and representation theory. Lecture 1
10:50 - 11:10	Coffee break
11:10 - 12:00	Max Wakefield (United States Naval Academy, USA) Matroids, arrangements, and representation theory. Lecture 2
12:10 - 14:00	Lunch
Afternoon	
14:00 - 14:50	Clément Dupont (University of Montpellier, France) On the monodromy of Milnor fibers of hyperplane arrangements. Lecture 3
14:50 - 15:10	Coffee break
15:10 - 16:00	<b>Graham Denham</b> (University of Western Ontario, Canada) Arrangements, wonderful models and toric varieties Lecture 3

16:00 - 16:45 **Tutorial session** 

#### Wednesday, March 13, 2019

#### Morning

- 09:00 10:50 **Daniel Cohen**(Louisiana State University, USA) On arrangement groups and associated invariants Lecture 3 and 4
- 10:50 11:10 **Coffee break**
- 11:10 12:00 **Graham Denham** (University of Western Ontario, Canada) Arrangements, wonderful models and toric varieties Lecture 4
- 12:00 14:00 Lunch

#### Afternoon

- 14:00 14:50 **Graham Denham** (University of Western Ontario, Canada) Arrangements, wonderful models and toric varieties Lecture 5
- 14:50 15:10 Coffee break
- 15:10 16:00 Max Wakefield (United States Naval Academy, USA) Matroids, arrangements, and representation theory. Lecture 3
- 16:00 16:45 **Christin Bibby** (University of Michigan, USA) Torics Arrangements. Lecture 1

#### Thursday, March 14, 2019

#### Morning

09:00 - 09:50	Max Wakefield (United States Naval Academy, USA) Matroids, arrangements, and representation theory. Lecture 4
10:00 - 10:50	<b>Clément Dupont</b> (University of Montpellier, France) On the monodromy of Milnor fibers of hyperplane arrangements. Lecture 4
10:50 - 11:10	Coffee break
11:10 - 12:00	<b>Clément Dupont</b> (University of Montpellier, France) On the monodromy of Milnor fibers of hyperplane arrangements. Lecture 5
12:10 - 14:00	Lunch
Afternoon	
14:00 - 14:50	<b>Christin Bibby</b> (University of Michigan, USA) <i>Torics Arrangements. Lecture 2</i>
14:50 - 15:10	Coffee break

- 15:10 16:00 Christin Bibby (University of Michigan, USA) Torics Arrangements. Lecture 3
- 16:00 16:45 **Tutorial session**

#### Friday, March 15, 2019

#### Morning

- 09:00 10:50 **Clément Dupont** (University of Montpellier, France) On the monodromy of Milnor fibers of hyperplane arrangements. Lecture 6 and 7
- 10:50 11:10 **Coffee break**
- 11:10 12:00 **Daniel Cohen**(Louisiana State University, USA) On arrangement groups and associated invariants Lecture 5
- 12:00 14:00 Lunch

#### Afternoon

- 14:00 14:50 **Daniel Cohen**(Louisiana State University, USA) On arrangement groups and associated invariants Lecture 6
- 14:50 15:10 Coffee break
- 15:10 16:00 Max Wakefield (United States Naval Academy, USA) Matroids, arrangements, and representation theory. Lecture 5
- 16:00 16:45 **Tutorial session**

Saturday and Sunday, March 16-17, 2019

Ha Long Bay Tour

## Monday, March 18, 2019

## Morning

09:00 - 09:50	Masahiko Yoshinaga (Hokkaido University) On free hyperplane arrangements and Terao's conjecture. Lecture 1
10:00 - 10:50	<b>Christin Bibby</b> (University of Michigan, USA) <i>Torics Arrangements. Lecture 4</i>
10:50 - 11:10	Coffee break
11:10 - 12:00	<b>Christin Bibby</b> (University of Michigan, USA) <i>Torics Arrangements. Lecture 5</i>
12:10 - 14:00	Lunch
Afternoon	
14:00 - 14:50	<b>Graham Denham</b> (University of Western Ontario, Canada) Arrangements, wonderful models and toric varieties Lecture 6
14:50 - 15:10	Coffee break
15:10 - 16:00	<b>Graham Denham</b> (University of Western Ontario, Canada) Arrangements, wonderful models and toric varieties

Lecture 7

18:30 - 20:30 School Party

#### Tuesday, March 19, 2019

#### Morning

09:00 - 10:50	Masahiko Yoshinaga (Hokkaido University)
	On free hyperplane arrangements and Teraos conjecture.
	Lecture 2 and 3
10:50 - 11:10	Coffee break

- 11:10 12:00 Max Wakefield (United States Naval Academy, USA) Matroids, arrangements, and representation theory. Lecture 6
- 12:00 14:00 Lunch

#### Afternoon

- 14:00 14:50 **Masahiko Yoshinaga** (Hokkaido University) On free hyperplane arrangements and Teraos conjecture. Lecture 4
- 14:50 15:10 **Coffee break**
- 15:10 16:00 Christin Bibby (University of Michigan, USA) Torics Arrangements. Lecture 6
- 16:00 16:45 **Tutorial session**

#### Wednesday, March 20, 2019

#### Morning

09:00 - 09:50	<b>Christin Bibby</b> (University of Michigan, USA) <i>Torics Arrangements. Lecture</i> 7
10:00 - 10:50	Masahiko Yoshinaga (Hokkaido University) On free hyperplane arrangements and Teraos conjecture. Lecture 5
10:50 - 11:10	Coffee break
11:10 - 12:00	Masahiko Yoshinaga (Hokkaido University) On free hyperplane arrangements and Teraos conjecture. Lecture 6

12:10 - 14:00 Lunch

#### Afternoon

- 14:00 14:50 **Daniel Cohen**(Louisiana State University, USA) On arrangement groups and associated invariants Lecture 7
- 14:50 15:10 **Coffee break**
- 15:10 16:00 Max Wakefield (United States Naval Academy, USA) Matroids, arrangements, and representation theory. Lecture 7
- 16:00 16:45 Masahiko Yoshinaga (Hokkaido University)
  On free hyperplane arrangements and Teraos conjecture. Lecture 7

## ABSTRACT OF COURSES

#### Course 1: On the monodromy of Milnor fibers of hyperplane arrangements

Clément Dupont Department of Mathematics, University of Montpellier Montpellier, France

Abstract: This is an introductory course, related to the monodromy operators and Milnor fiber of an arrangement. The Milnor fiber F and the monodromy operators  $h^m: H^m(F; \mathbb{C}) \longrightarrow H^m(F; \mathbb{C})$  will be carefully defined, and the main examples discussed in detail. Several approaches for the computation of the Betti numbers  $b_i(F)$  and of the eigenvalues of the monodromy operators hm will be discussed as well.

#### Course 2: Arrangements, wonderful models and toric varieties

Graham Denham Department of Mathematics, University of Western Ontario Ontario, Canada

Abstract: This is an introductory course centred around some geometric aspects of complex hyperplane arrangements. By viewing the complement  $M(\mathcal{A})$  as a subvariety of a complex torus in a toric variety, we see a number of interesting spaces constructed from a hyperplane arrangement, such as De Concini-Procesi's wonderful compactification. The course will give a working introduction to the combinatorics of matroids and of toric varieties, leading to the notion of the Bergman fan and a tropical linear space. We will consider some modern applications.

#### Course 3: On free hyperplane arrangements and Terao's conjecture

Masahiko Yoshinaga Department of Mathematics, Hokkaido University Sapporo, Japan

Abstract: This is an introductory course related to the Terao's conjecture. The notion of a free singularity and of a free hypersurface will be introduced, and a special attention will be given to the analogies and differences between the local analytic case and the global graded algebraic case. The case of line arrangements in  $\mathbb{P}^2$  will be discussed in detail, in particular the cases of no more than 13 lines, when the conjecture is known to hold.

### Course 4: On arrangement groups and associated invariants Daniel Cohen Department of Mathematics, Louisiana State University Louisiana, USA

*Abstract:* This is an inductory course discussing "arrangement groups", fundamental groups of complements of complex hyperplane arrangements, various invariants of these groups, and the interplay among them.

#### Course 5: Matroids, arrangements, and representation theory

Max Wakefield Department of Mathematics, United States Naval Academy Annapolis, USA

Abstract: This is an introductory course, centered on two aspects, and related to problem of Kazhdan-Lusztig polynomials. First, the combinatorial natural of most of the questions in this theory, which is best formalized by using the matroids. Then, the fact that many important classes of hyperplane arrangements come from complex reflection groups acting on a vector space. Indeed, the hyperplanes of the arrangements are just the reflecting hyperplanes of the group action. A discussion of the Braid Arrangement, of the Monomial Arrangement and of the Full Monomial Arrangement will illustrate this aspect.

#### **Course 6: Toric arrangements**

Christin Bibby Department of Mathematics, University of Michigan Ann Arbor, USA

Abstract: This is an advanced course in which instead of looking at hyperplanes in an affine space  $\mathbb{C}^n$  or in a projective space  $\mathbb{P}^n$ , ones looks at affine subtori in an algebraic torus  $(\mathbb{C}^*)^n$  or at abelian subvarieties (say subtori in an abelian variety A, say an algebraic compact torus). Many results from the case of hyperplane arrangements extend to these new situations, but sometimes new techniques and new ideas are involved.

# WORKSHOP PROGRAM

#### Thursday, March 21, 2019

#### Morning

10:00 - 10:50	<b>Alexandru Dimca</b> (University of Nice Sophia Antipolis) Tensor decomposition and hyperplane arrangements
11:10 - 12:00	<b>Takahiro Nagaoka</b> (Kyoto University) Hypertoric varieties and hyperplane arrangements
12:00 - 14:00	Lunch

### Afternoon

14:00 - 14:50	<b>Tatsuya Horiguchi</b> (Osaka University) A basis of the cohomology ring of a regular nilpotent Hessenberg variety
15:10 - 16:00	<b>Tran Nhat Tan</b> (Hokkaido University) A combinatorial description of the exponents of $A_1^2$ restrictions

## of Weyl arrangements

#### Friday, March 22, 2019

#### Morning

10:00 - 10:50	Takuro Abe (Kyushu University)
	Combinatorics of the addition-deletion theorems for
	free arrangements

- 11:10 12:00 Norihiro Nakashima (Nagoya Institute of Technology) Coboundary polynomials of Coxeter arrangements and Catalan arrangements
- 12:00 14:00 Lunch

#### Afternoon

14:00 - 14:50	Masahiko Yoshinaga (Hokkaido University)
	Double coverings of arrangement complements and 2-torsion in
	Milnor fiber homology

15:10 - 16:00 Alex Suciu (Northeastern University) Arrangement groups, lower central series, and Massey products

# WORKSHOP ABSTRACT

#### Tensor decomposition and hyperplane arrangements

Alexandru Dimca Laboratoire J.-A. Dieudonné, Université de Nice- Sophia Antipolis Nice, France

Abstract: In this talk we give information on symmetric tensors in n variables of Waring rank n + 1.

Hypertoric varieties and hyperplane arrangements Takahiro Nagaoka Kyoto University, Kyoto, Japan

*Abstract:* Hypertoric varieties are algebraic varieties, defined as an analogue of toric varieties. As the geometric properties of (projective) toric varieties can be studied by the associated polytopes, hypertoric varieties can be studied by its associated hyperplane arrangements. In this talk, I will introduce hypertoric varieties with examples and pictures. Then, I will discuss the classification of singularities of affine hypertoric varieties and counting its good resolutions in terms of associated hyperplane arrangements.

### A basis of the cohomology ring of a regular nilpotent Hessenberg variety Tatsuya Horiguchi

Osaka University, Osaka, Japan

Abstract: Hessenberg varieties are subvarieties of a full flag variety. This subject lies at the intersection of, and makes connections between, many research areas such as algebraic geometry and topology, representation theory, and combinatorics. In particular, the cohomology ring of a regular nilpotent Hessenberg variety can be described by the logarithmic derivation module of the ideal arrangement. In this talk, I would like to explain a basis of the cohomology ring of a regular nilpotent Hessenberg variety in terms of the root system in type A. This is joint work with Makoto Enokizono, Takahiro Nagaoka, Akiyoshi Tsuchiya.

# A combinatorial description of the exponents of $A_1^2$ restrictions of Weyl arrangements

Tan Nhat Tran

Hokkaido University Sapporo, Japan

Abstract: Let  $\mathcal{A}$  be a Weyl arrangement in an  $\ell$ -dimensional Euclidean space. Using a case-by-case argument, Orlik-Terao (1993) proved that any restriction of  $\mathcal{A}$  is free. Prior to this, Orlik-Solomon (1983) had completely determined the exponents of these arrangements by exhaustion. However, describing theoretically their exponents is still a difficult task. A classical result, due to Orlik-Solomon-Terao (1986), asserts that the exponents of any  $A_1$  restriction i.e., the restriction of  $\mathcal{A}$  to a hyperplane, are given by  $\{m_1, \ldots, m_{\ell-1}\}$ , where  $\exp(\mathcal{A}) = \{m_1, \ldots, m_\ell\}$  with  $m_1 \leq \ldots \leq m_\ell$ . As a next step after Orlik-Solomon-Terao towards understanding the exponents of restrictions, we are especially doing the investigation on the  $A_1^2$  restrictions i.e., the restrictions of  $\mathcal{A}$  to subspaces X of the type  $A_1^2$ . In this talk, we will present a description of the exponents of such restrictions in terms of the classical notion of related roots by Kostant (1955). This is a joint work with Takuro Abe and Hiroaki Terao.

# Takuro Abe: Combinatorics of the addition-deletion theorems for free arrangements

Takuro Abe Institute of Mathematics for Industry Kyushu University Kyushu, Japan

*Abstract:* The most useful result to check/prove the freeness of arrangements is Terao's addition-deletion theorem. We show that this is combinatorial. Namely, if you are given an arrangement and its addition/deletion, then whether they are free or not depends only on the intersection lattice. Based on them, we introduce two classes of free arrangements called the divisionally and additionally free arrangements in which Terao's conjecture is true.

#### Coboundary polynomials of Coxeter arrangements and Catalan arrangements

Norihiro Nakashima

Nagoya Institute of Technology Nagoya, Japan

Abstract: The Tutte polynomial gives us many interesting information of graphs and hyperplane arrangements. In particular the characteristic polynomial can be computed by this polynomial. Crapo introduced the coboundary polynomial for a matroid, which is essentially equivalent to the Tutte polynomial. Also the Hamming weight enumerator for the matroid of an error correcting code is transformed from the coboundary polynomial. In this talk I present a computation of coboundary polynomials for the matroids of Coxeter arrangements and Catalan arrangements. This is join work with S. Tsujie.

#### Double coverings of arrangement complements and 2-torsion in Milnor fiber homology

Masahiko Yoshinaga Hokkaido university, Sapporo, Japan

Abstract: We prove that mod 2 Betti numbers of double coverings of arrangement complements are combinatorially determined. Applying this result to the icosidodecahedral arrangement (which is an arrangement of 16 planes in  $\mathbb{R}^3$  related to the icosidodecahedron) we conclude that the first homology group of its Milnor fiber has 2-torsion.

## Arrangement groups, lower central series, and Massey products Alex Suciu Northeastern University, Boston, USA

*Abstract:* I will discuss some recent advances in our understanding of fundamental groups of complements of complex hyperplane arrangements, with emphasis on associated graded and holonomy Lie algebras, as well as Massey products in positive characteristic. The talk will be based on current joint work with Rick Porter.

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