Ordinary Differential Equations

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Syllabus

1. Introduction (4 hours)
   (a) Examples of ODEs
   (b) Solution techniques for simple problems:
      i. Separation of variables
      ii. Integrating factors for first-order linear equations
2. Numerical methods (3 hours)
   (a) Euler's method
   (b) The classical fourth-order Runge-Kutta method
3. Systems of first-order linear ODEs (12 hours)
   (a) Basis theory (existence and uniqueness, dimension of solution space, Wronskian test for linear independence)
   (b) Solution of homogeneous systems using eigenvalues and eigenvectors (real eigenvalues, complex eigenvalues, defective matrices); fundamental matrices and the matrix exponential
   (c) Inhomogeneous systems and variation of parameters
4. Scalar equations with constant coefficients (5 hours)
   (a) The auxiliary equation
   (b) Reduction of order
   (c) Inhomogeneous equations (variation of parameters and the method of undetermined coefficients)
5. Series solutions of linear ODEs (10 hours)
   (a) Solutions near an ordinary point
   (b) Solutions near a regular singular point
   (c) Bessel's equation
6. Nonlinear ODEs and phase plane analysis (7 hours)

Examinations Four hours are reserved for examinations (two one-hour exams and a two-hour final exam).

Applications The following applications will be incorporated in the lectures: Radioactive decay, falling objects (with and without air resistance), population dynamics (exponential growth, logistic growth, the effect of immigration, competing species, and predator-prey dynamics), mass-spring systems (both single-mass/single-spring and multiple-masses/multiple-springs systems), LRC circuits (including circuits with a variable capacitor), the simple pendulum.