

**Final report on the CIMPA school on:**  
**Elliptic curves: arithmetic and computation**  
**Universidad de la República, Montevideo, Uruguay.**  
**February 11-22, 2019**



The CIMPA research school "Elliptic curves: arithmetic and computation" was organized by CIMPA in conjunction with the Universidad de la República, Montevideo, It was held at the Universidad de la República, Montevideo, from the 11th to the 22th of February 2019. The website of the school with updated information is located at

<http://www.rnta.eu/Montevideo2019/>

The classes were given by 9 lecturers from 4 different countries including 5 from Italy, 2 from France, and one each from the Netherlands and Uruguay, there were 2 female lecturers which is 22.2% of the total number of lecturers.

There were 57 participants from 13 countries including 14 from Uruguay, 9 each from Argentina and Colombia, 8 from Chile, 5 from Brazil, 3 each from Croatia and Mexico, 2 each from Peru and French Polynesia, and 1 each from Poland and France, the last participant was the representative of CIMPA coming from Spain.

The school obtained supports from CIMPA, the International Center for Theoretical Physics, Foundation Compositio Mathematica, the Italian minister of foreign affairs, the commission for developing countries of the International Mathematical Union (IMU-CDC), the national research group G.N.S.A.G.A. of the Istituto Nazionale di Alta Matematica, ANII - Proyecto FCE 2017 136609, CMat (Facultad de Ciencias), Pedeciba Matemática, Facultad de Ingeniería, IMERL (Facultad de Ingeniería), CSIC (Universidad de la República), French Embassy, and LIA-IFUM (Instituto Franco-Uruguayo de Matemática), and was organized in collaboration with the Roman Number Theory Association. Moreover the travel of 2 students from IMPA was subsidised by IMPA itself, and the travel of one student from Chile and was Brasil was subsidised by research grants so all the participants from South America were fully funded.

The scientific program of the school was very interconnected as one of the main goals of the school was to develop the theory of elliptic curves from its very beginning to the level

needed to understand and appreciate the Birch and Swinnerton Dyer conjecture. Another aspect that we dwell on was that of explicit computations, in particular there were several classes on computation with the PARI/GP software. All the speakers made a great effort to give clear talks, and they succeeded very well. We add two extra session of exercises in the second week and students were very eager to present solutions to the problems assigned. In general the students interacted greatly with the lecturers asking many questions after classes and during breaks.



Over the weekend an excursions took place on Saturday. We climbed the Cerro Pan Azúcar the third tallest peak of Uruguay. It was a good occasion for social interaction and for students to ask question about future schools and programs and fellowships to study in Europe.



Participants were hosted in a hostel nearby the venue of the conference and took all their meals in the cafeteria of the university. The hostel had several common spaces with tables which were ideal for studying. Moreover the hostel was occupied only by the participants of the school and hence there was plenty of time and occasion for interaction of the students, and indeed the students interacted greatly during the school. We had a social dinner on the Wednesday of the second week.

Below you find a the members of the various committees, the program of the school, the list of participants, and the detailed budget.

*Scientific Committee*

Christophe Ritzenthaler (Université de Rennes 1), Elisa Lorenzo Garcia (Université de Rennes 1), Laura Geatti (Università di Roma Tor Vergata), René Schoof (Università di Roma Tor Vergata), Valerio Talamanca (Università Roma Tre), Gonzalo Tornaria (Universidad de la República)

*Local Committee*

Gonzalo Tornaria (Universidad de la República), Gustavo Rama (Universidad de la República), Nicolás Sirolli (Universidad de Buenos Aires), Claudio Qureshi (Universidad de la República).

*Courses*

**Introduction to algebraic curves**

Christophe Ritzenthaler

**Elliptic curves**

Peter Stevenhagen

**The Birch and Swinnerton-Dyer conjecture**

René Schoof

**Elliptic curves algorithms**

Laura Geatti

**Heights on elliptic curves**

Valerio Talamanca

**Introduction to Elliptic curves**

Elisa Lorenzo Garcia

**The Mordell-Weil Theorem**

Fernando Rodriguez Villegas

**L-functions and PARI/GP Tutorials**

Francesco Pappalardi

**Modular forms and Elliptic curves**

Gonzalo Tornaria

Schedule

Week 1					
	Monday	Tuesday	Wednesday	Thursday	Friday
09.00-09.50	Registration & Opening Ceremony	Algebraic curves Christophe Ritzenthaler	Algebraic curves Christophe Ritzenthaler	Elliptic curves Elisa Lorenzo Garcia	The Weak Mordell-Weil theorem Fernando Rodriguez Villegas
10.00-10.50					
10.50-11.10	BREAK				
11.10-12.00	Algebraic curves Christophe Ritzenthaler	Elliptic curves Elisa Lorenzo Garcia	Elliptic curves Elisa Lorenzo Garcia	Diophantine equations and elliptic curves Fernando Rodriguez Villegas	Elliptic curves over Local fields René Schoof
12.10-13.00					
13.00-15.00	LUNCH				
15.00-15.50	Galois cohomology Fernando Rodriguez Villegas	Complex elliptic curves Peter Stevenhagen	Free Afternoon	Elliptic curves over finite fields Peter Stevenhagen	Explicit 2-descent Peter Stevenhagen
16.00-16.50					

  

Week 2					
	Monday	Tuesday	Wednesday	Thursday	Friday
09.00-09.50	Modular forms and elliptic curves Gonzalo Tornaria	The Mordell-Weil Theorem Valerio Talamanca	Modular forms and elliptic curves Gonzalo Tornaria	Canonical heights Valerio Talamanca	Elliptic curves algorithms Laura Geatti
10.00-10.50					
10.50-11.10	BREAK				
11.10-12.00	Heights on projective spaces Valerio Talamanca	Elliptic curves algorithms Laura Geatti	L-functions Francesco Pappalardi	Elliptic curves algorithms Laura Geatti	The Birch&Swinnerton-Dyer conjecture René Schoof
12.10-13.00					
13.00-15.00	LUNCH				
15.00-15.50	Computations with GP/PARI Francesco Pappalardi	Computations with GP/PARI Francesco Pappalardi	Free Afternoon	Image of Galois René Schoof	Modular forms and elliptic curves Gonzalo Tornaria
16.00-16.50					
16.50-17.20		Merienda		Merienda	
17.20-19.00		Exercises session		Exercises session	

# Participants

Santiago Arango, *Instituto de Matemática Pura e Aplicada*, Rio De Janeiro, Brazil  
Diego Armentano *Universidad de la República*, Uruguay  
Cristian Baeza *Pontificia Universidad Católica de Chile*, Chile  
Leandro Bentancur , *Universidad de la República*, Uruguay  
Gaetan Bisson, *Tahiti university* French Polynesia Patricia Borges dos Santos, *Universidade Federal de Uberlândia* Brazil  
André Filipe Braga Lelis, *Federal University of Rio de Janeiro* Brazil  
Gustavo Andrés Cabaña, *Universidad Nacional del Litoral*, Santa Fe Argentina  
Nichool Castañeda, *Universidad del Valle*, Colombia  
Alberto Allonso Castillo García, *Pontificia universidad católica del Peru*, Peru  
Bilson Castro, *Universidad de Antioquia*, Colombia  
Christian Caticha, *Universidad de la República*, Uruguay  
María de los Angeles Chara, *Universidad Nacional del Litoral*, Argentina  
Paula Mercedes Chiapparoli *Universidad Nacional de Córdoba*, Argentina  
Luis Alberto Cid Carreño *Universidad de Talca*, Chile  
Marcel de Sena Dall Agnol *Instituto de Matemática Pura e Aplicada*, Brazil  
Luis Miguel Delgado Ordoñez *Universidad del Cauca*, Popayan., Colombia  
Harold Steven Erazo Jiménez *Universidad del Valle*, Colombia  
Fernando José Figueroa *Pontificia Universidad Católica de Chile* ,Chile  
Irving-Josue Flores-Romero *Politécnico Nacional*, Mexico  
Hernán Galletti *Universidad de Buenos Aires*, Argentina  
Alejo García *Universidad de la República*, Uruguay  
Sofia Garzon Mora *Universidad de los Andes*, Bogota' Colombia  
Laura Geatti *Università di Roma "Tor Vergata"*, Italy  
Josefina Gonzalez *Universidad de la República* Uruguay  
Pedro Adiel Hernandez Leal, *Instituto Politécnico Nacional*, San Pedro Zacatenco Mexico  
José de Jesús Hernández Serda, *Universidad Nacional Autónoma de México*, Cuernavaca, Mexico  
David Jaramillo, *Universidad de los Andes*, Bogota Colombia  
Ivan Krijan, *University of Zagreb* Croatia  
Joaquín Lejtregger, *Universidad de la República* Uruguay  
Joaquín Lema, *Universidad de la República* Uruguay  
Dinamarca Leonardo, *IMPA*, Rio de Janeiro Chile  
PawelLewulis, *Polish academy of sciences* Poland  
Elisa Lorenzo Garcia *Université de Rennes 1*, France  
Gonzalo Esteban Manzano Flores *Universidad de Santiago de Chile*, Chile  
Pablo Maurente *Universidad de la República*, Uruguay  
Regis Monneau *Ecole des ponts Paristech*, France  
Fernando Montans *Universidad de la República*, Uruguay  
Marcos Morales *Pontificia Universidad Católica*, Chile  
Jorge Mozo *Universidad de Valladolid*, España  
Nicolas Fitzgerald Munoz Herrera *Universidade Federal de Uberlandia*, Brazil  
Horacio Navarro Oyola *Universidad del Valle*, Colombia  
Roger Oyono *Tahiti university*, French Polynesia  
Francesco Pappalardi *Università Roma Tre*, Italy  
Juan Francisco Piombo *Universidad de Buenos Aires*, Argentina  
Claudio Qureshi *Universidad de la República* , Uruguay  
Santiago Radi *Universidad de la República*, Uruguay  
Gustavo Rama *Universidad de la República*, Uruguay  
Andrés Mauricio Ramírez Pérez *Universidad Sergio Arboleda*, Colombia



Chrisophe Ritzenthaler *Université de Rennes 1* France  
Fernando Rodriguez Villegas *ICTP*, Italy  
Jorge Luis Rojas Orbegoso *Pontificia universidad católica del Peru*, Peru  
Lucia Rossi *Universidad Nacional de Mar del Plata*, Argentina  
Hamilton Ruiz *Universidad del Cauca, Popayán*, Colombia  
Bruno Scarone *Universidad de la República*, Uruguay  
René Schoof *Università di Roma "Tor Vergata"*, Italy  
Tomás Seguel *Universidad de Santiago de Chile*, Chile  
Nicolás Sirolli *Universidad de Buenos Aires*, Argentina  
Peter Stevenhagen *Universiteit Leiden*, The Netherlands  
Rolando Alfredo Suárez Quevedo *Universidad de Talca*, Chile  
Valerio Talamanca *Università Roma Tre*, Italy  
Gonzalo Tornarí *Universidad de la República*, Uruguay  
Ricardo Toso *Universidade Federal do Rio de Janeiro*, Brazil  
Antonela Trbovic *University of Zagreb*, Croatia  
Lucas Villagra Torcomian *Universidad Nacional de Córdoba*, Argentina  
Angel Dario Villanueva *Universidad Nacional de Córdoba*, Argentina  
Borna Vukorepa *University of Zagreb*, Croatia