Scientific report: CIMPA school "Hopf Algebras and Tensor Categories"

The school was organized around two thematic axes: Hopf algebras and tensor categories. The school was intended to introduce Ph.D. students and young researchers on both areas, to explain how they are interrelated, and also to present applications and new lines of research.

The courses on Hopf algebras aim to present the basics of this theory, to present the most relevant techniques and the classification program of finite-dimensional Hopf algebras. We provided an introduction to the basic definitions and examples of this theory, as well as the latest tools developed in the area, with an emphasis in the theory of Nichols algebras. We wanted to show how this theory can be applied to other areas of mathematics, and also its interrelation with the theory of tensor categories.

The courses on tensor categories aim to present the basic notions of fusion tensor categories, some new developments in the classification of such objects, and how these algebraic objects appear as symmetries of diverse structures of mathematical physics and computing science. The courses surveyed an introduction, some techniques applied in their classification and also how the theory of Hopf algebras can be a powerful tool in this context.

The school was organized at the «Hotel del Lago» (http://hoteldellago.com.ar) in the city of La Falda, a state of Córdoba, Argentina. This hotel has a long history of organizing scientific encounters; the staff is prepared for this type of events and they were very helpful. The school was organized to coincide with a national encounter of algebra (IX elENA), see webpagehttp://elenaix.dm.uba.ar/. All participants and speakers were staying at the hotel. conferences and courses were given also in the same hotel. Organizing both encounters at the same time in the same place allowed us to connect students with foreing and national researchers, to give an academic environment and also to find a better price for lodging. Students from CIMPA school could attend to research conferences presented at elENA and also some participants from elENA attended to courses given at CIMPA.

For the school we mainly had three goals:

- Introduce to students in the area of (pointed) Hopf algebras and tensor categories,
- connect researches with undergraduate students from Argentina who want to pursue doctoral studies,
- connect Ph.D Students with researchers to expand their lines of research.

All goals were achived.

There were six courses in the school:

1. Gastón García from Universidad Nacional de la Plata, Argentina: "**Introduction to Hopf algebras"**.

Abstract of the course: The course intended to introduce students in the theory of Hopf algebras. We introduced the notions of algebras, coalgebras, bialgebras and Hopf algebras. We showed relevant examples coming from the theory of Lie algebras. All invariants of Hopf algebras were explained; such as the coradical, the coradical

filtration, the space of primitive elements and the group of group-like elements. The theory of Hopf modules was explained, and the definition of integrals for Hopf algebras given.

2. Iván Angiono from Universidad Nacional de Córdoba, Argentina: **"The lifting method"**

Abstract of the course: I this course the method introduced by Andruskiewitsch and Schneider was explained, to classify finite-dimensional pointed Hopf algebras. We showed the relation with pointed Hopf algebras and Nichols algebras, and how powerful this method can be. Explicit examples were presented.

3. Istvan Heckenberger from Philipps Universiy of Marburg, Germany: **"Introduction to Nichols algebras".**

Abstract of the course: Nichols algebras, discovered by Nichols in the 70s, have gained relevance since their emergence as fundamental invariant for the classification of pointed Hopf algebras. Given a braided vector space V (e.g. a solution of the braid equation), a Nichols algebra is an associative algebra constructed from V with a universal property making it a Hopf algebra in a braided category.

The objectives of this course are the following:

- to introduce Nichols algebras, starting from the definition of braided vector spaces;
- to give different constructions of Nichols algebras;
- to discuss several examples and propose different computations for simple cases;
- to describe some families of Nichols algebras, mainly of diagonal type, with the main properties of each one of them

4. Julia Plavnik from Indiana University, USA: "**An introduction to tensor and modular categories.**"

Abstract of the course: The problem of classifying modular categories is motivated by applications to topological quantum computation as algebraic models for topological phases of matter. These categories have also applications in different areas of mathematics like topological quantum field theory, von Neumann algebras, representation theory, and others. In this mini-course, we started by introducing some of the basic definitions and properties of tensor, fusion, braided, and modular categories. We gave some concrete examples to have a better understanding of their structures. We also presented the main properties of modular categories, such as its connection with the modular group, rank-finiteness, among others. Then we will give an overview of the current situation of the classification program for modular categories. We will explain some of the techniques that we found useful to push further the classification. On doing this, we will explore some of the interesting constructions that give rise to modular categories from known ones.

5. Sergey Neshveyev from Mathematics institute, University of Oslo, Norway: "**Compact quantum groups"**.

Abstract of the course: Compact quantum groups is a particular class of Hopf algebras with involution introduced by Woronowicz in the 1980s. On the one hand, the involution simplifies some algebraic considerations; for example, the Hopf algebras we get are automatically cosemisimple. On the other hand, it poses new problems and motivates introduction of various notions related to analysis, such as, for example, amenability and various approximation properties. After a brief introduction into the subject, the goal of the lectures is to present several

results whose proofs rely on a combination of purely algebraic and analytic methods.

6. Chelsea Walton from Temple University, USA: "**Ring-theoretic and homological properties of Hopf algebras and (co)module algebras."**

Abstract of the course : I will provide a survey of many homological and ring-theoretic techniques used to study Hopf algebras, both in the finite dimensional and infinite dimensional case, and their actions on noncommutative algebras with good properties (e.g. Artin-Schelter regular algebras).

The Programm of the school was the following:

Time	Monday 29	Tuesday 30	Wednesday 31	Thursday 1	Friday 2	Saturday 3	Sunday 4	Monday 5
9:00 - - 10:00	Registration	García	García	Walton	Walton	*Free Day*	García	Plavnik
10:00 - 10:15	Pause							
10: 15 - 11:15	García	Angiono	Angiono	Heckenberger	Neshveyev	*Free Day*	Plavnik	Plavnik
11:15 - 11:30	Coffee break							
11:30 - 12:30	Conference	Heckenberger's Conference	Conference	Conference	Conference	*Free Day*	Neshveyev	Neshveyev
12:40 - 14:30	Lunch							
14:30 - 15:15	García	Angiono	Heckenberger	Neshveyev	Heckenberger	*Free Day*	Neshveyev	Neshveyev
15:15 - 15:30	Pause							
15:30 - 16:30	Angiono	García	Angiono	Angiono	Plavnik	*Free Day*	Plavnik	
16:30 - 16:45	Coffee break							
16:45 - 18:00	Walton	Heckenberger	Walton	Walton	Heckenberger	*Free Day*	Plavnik	