# Links between mathematics and engineering in 19th century kinematic models collection

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In November 2003, the American Society of Mechanical Engineers (ASME) designated the *Cornell Reuleaux Kinematic Model Collection* as a National Historic Landmark Collection. It took another year until the official designation ceremony was possible in newly opened Duffield Hall that now is the permanent display place of this collection. What is this collection and how it is connected with mathematics?

In the late 19th century there were two contrasting approaches to the education of mechanical engineers in US. As an example of the complexities of the school-versus-shop controversy in mechanical engineering education, Monte A. Calvert in [1] discussed the case of Sibley College of Engineering at Cornell University, which had in succession, leading proponents of both approaches, John Edson Sweet (1832-1916) and Robert Henry Thurston (1839-1903). Sweet started to work in Sibley College in 1873. His idea of education was to have a master craftsman surrounded by humble, eager, admiring apprentices, learning about mechanics, industry, and life. He hated the routine and standardized nature of much of educational practice, and spoke of "examinations and diplomas" as the two great evils of schools and colleges. He said:

> It is not always the uneducated, the insane or the stupid, who produce failures, nor the best educated, most thoughtful, or most experienced who bring out everything according to the original intention. The unexpected comes to good and bad alike, and so in our teachings to the young and our planning for ourselves, is it not well to have our statements and our speculations pretty well saturated with the elements of uncertainty? (ASME, Trans., VII (1885-86), 156)

It is known that Sweet acquired a small but devoted group of students, who literally worshipped him. But, in general, Cornell's mechanical education was criticized in technical journals. Sweet became disgusted and quit in a huff in 1879. The first president of Cornell University, A.D. White, in 1882 made an effort to answer charges pointing out that Sibley College had two professors and much new equipment. Part of this new of equipment was 266 Reuleaux kinematic models, acquired in 1882 with an $8,000 gift from Hiram Sibley of Rochester, an early university benefactor. In 1885 Robert H. Thurston came to Sibley College and he went to work at once to raise entrance requirements in order to eliminate much of the elementary work in the first two years of the curriculum. The curriculum was reorganized to include basic science, higher mathematics, and language in the first two years. His aim was to perfect his model of the pure technical school, and, as he wrote to A.D. White in 1885:

the more rapidly the lower portion of the work is forced back into the preparatory and lower schools, and the more colleges reach upwards into the higher fields of investigation, the more they will accomplish for the world.

For more about entrance requirements and teaching mathematics at the end of 19th century in Cornell University, see [12].


***Figure 1: Cover of* Scientific American**

In 1880's *Scientific American* was "a weekly journal of practical information, art, science, mechanics, chemistry, and manufactures". On its October 17, 1885 issue cover (Fig. 1) was featured Sibley College that had become at that time one of the great engineering schools in United States. One of these pictures shows a room that was called "museum of mechanisms" (Fig. 2).

***Figure 2: Reuleaux models in Sibley Hall***

This museum of mechanisms was a newly acquired kinematic model collection that was designed by famous German engineer, Prof. Franz Reuleaux (1829-1905), and made by his friend, German instrument maker Gustav Voigt. Reuleaux believed that there were scientific principles behind invention and the creation of new machines, what we call synthesis today. This belief in the primacy of scientific principles in the theory and design of machines became the hallmark of his worldwide reputation, particularly in the subject of machine kinematics [6].

***Figure 3: Franz Reuleaux (1829-1905)***

Reuleaux was the first engineer to use topological ideas in kinematics. He also devoted serious attention to education and the role of mathematics. Reuleaux wrote:

> No student of the world's present state of general culture can have failed to observe potent influence that the technical sciences of our day are exerting, nor the extent to which they have fitted us for incomparably greater achievements than were possible a few centuries ago.
> … But all this is well known, and yet among the generality of educated people, and perhaps even within the narrower circle of professional men, it hardly seems to meet with the appreciation which it merits. The useful arts, scientifically developed, have not as yet been accorded a sufficiently extended and deserved recognition as a factor, and more – as a powerful lever in elevating the standard of general culture.
> … The forces of nature which advance taught us to look to for service are mechanical, physical and chemical; but the prerequisite to their utilization was a full equipment of mathematics and natural sciences. This entire apparatus we now apply, so to say, as a privilege.
> … The instruction in the polytechnic school has of necessity to adopt as fundamental principles the three natural sciences – mechanics, physics, and chemistry, and the all-measuring master art of mathematics. [9]

F. Reuleaux believed in the use of demonstration models to express mathematical and kinematic ideas. He built a large collection of 800 mechanism models in Berlin and marketed 350 of them to universities around the world. Unfortunately much of this collection was destroyed during World War II, but some originals and reproductions of these models can be found in the Deutsche Museum in Munich, the University of Hanover, Kyoto University, Moscow Bauman Technical School, Karlov University in Prague and possibly in some other places we do not know yet. But the largest collection of these models is in Cornell University that has 220 (from originally acquired 266) Reuleaux models. Since 2002 we are working on developing "Kinematic Models for Design Digital Library" and you can explore these models on our website [5]. Besides having still images of these models, there is historical information, interactive movies that allow exploring how these models work. We also have scanned rare books that are important in the history of technology. Significant part of this project is connecting models and mathematical ideas behind those models to use them in the classroom. We have developed teaching materials that can be found in the section called "Tutorials". In this section one can find also a list of biographies for names appearing in the historical descriptions of the models and authors of rare books.

Franz Reuleaux incorporated mathematics into design and invention of machines in his work *Kinematics of Machinery* [8]. For mathematicians the best known idea is "Reuleaux triangle" (Fig. 4), which is one of the curves with constant width.

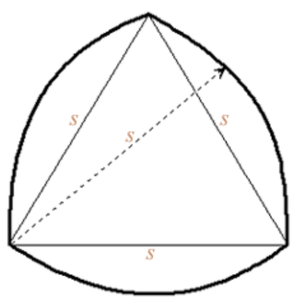


***Figure 4: Construction of Reuleaux triangle***

***Figure 5: Window of Notre Dame Cathedral, Bruges, Belgium***

This curved triangle can be seen in some gothic windows (Fig. 5), it also appears in some drawings of Leonardo da Vinci and Leonard Euler, but Reuleaux in his *Kinematics* gave the first complete analysis of such triangles, and he also noticed that such curves could be generated from any regular polygon with an odd number of sides. Reuleaux applied to such curves Poinsot's theory of rolling (centrodes, roulettes). We can see visually this analysis on one of Reuleaux models "curved triangle in the square" (Fig. 6).

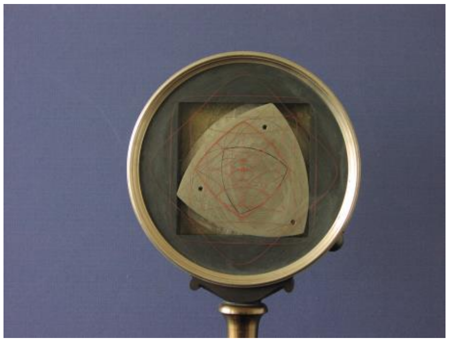

***Figure 6: Curved triangle in the square***

We have developed a material about Reuleaux triangle that can be used in school mathematics classes [14].

Many of the Reuleaux models show change in motion: circular to trigonometric (slider crank), circular to elliptical (double slider crank), circular to straight-line motion (straight-line mechanisms) — analysis of some of these motions involves calculus and inversive geometry (Peaucellier-Lipkin linkage). Gear mechanisms in the collection use properties of epicycloid and hypocycloid (Fig. 7).

***Figure 7: Epicycloidal gear design***

Several of Reuleaux models were included in Walter Dyck's *Katalog mathematischer und mathematisch-physikalischer Modelle, Apparate und Instrumente* [2]. Walter von Dyck (1856-1934) was one of the creators of the Deutsches Museum of Natural Science and Technology, and he was also appointed as the second Director of the Museum in 1906. The Deutsches Museum was first of its kind and its ideas were soon copied by other science museums around the world. For his catalogue Dyck chose from the Reuleaux models three that demonstrate properties of the cycloid on a sphere (Fig. 8).


***Figure 8: Rolling on the sphere***

Another mathematical application, which appears in the Reuleaux model collection, is the use of circle involutes in pumps and combining other geometrical figures into pumps [10]. Fig. 9 shows a pump that is constructed based on circle involute. This mechanism, for example, was used in Payton's water meter.


***Figure 9: Spiral pump***

Fig. 10 shows different designs of pumps, using geometrical properties.

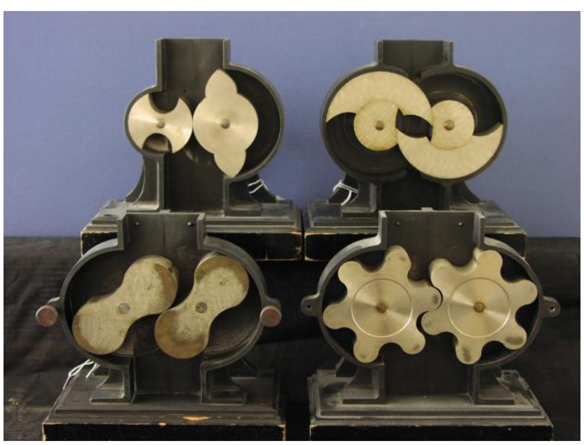
***Figure 10: Different pump designs***

Universal joint (Fig. 11) used to be a standard mechanism in all cars. There is a disagreement among historians of technology, whether it was discovered by G. Cardano (1501-1576) or R. Hooke (1635-1703); therefore it is still called also Cardano or Hooke's joint. But it is interesting that first patent on the universal joint (for using it in cars), was obtained by Clarence W. Spicer in 1903 while he was a student in Cornell University. Mathematical explanation of the change of the motion in the universal joint can be explained easily using spherical geometry (Fig. 12). For more details, see [15, 16].


***Figure 11: Universal joint***

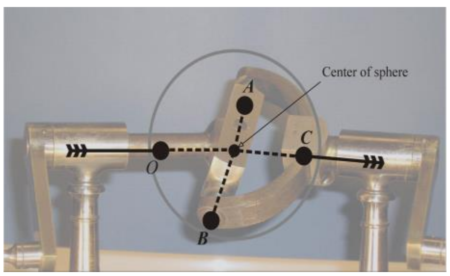


***Figure 12: Spherical geometry in the design of universal joint***

Some of Reuleaux mechanisms demonstrate effective use of Möbius band (Fig. 13).

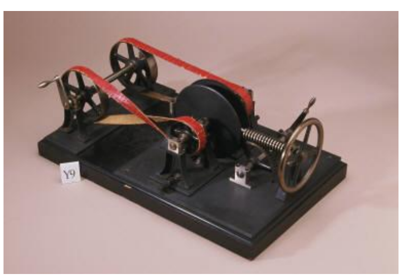

***Figure 13: Reversing belt transmission mechanism***

Reuleaux was classifying his mechanisms using alphabet, e.g., assigning letters to different groups of his mechanisms. That way he was stressing that each individual mechanism is like a letter in the alphabet and combining them together we are getting words and sentences or machines. His style of classification resembles later ideas used in topology and theoretical computer science. Largest number of mechanisms (39) is in so called S-series — straight line mechanisms. It has been a challenge in technology since ancient times — how to change a circular motion into straight-line motion [4]. This problem was essential for James Watt (1736-1819) when he was working on improving steam engine. In [13] we describe a short history of this problem and its solution in 19th century (Fig. 14). Mathematical aspects of the Peaucellier-Lipkin linkage involve inversive geometry that is described also in our tutorials [11]. There is renewed interest in design of different linkages, for example, in mathematics they are discussed in rigidity theory, in engineering linkages are widely used in robotics.

***Figure 14: Peaucellier-Lipkin linkage***

But it is interesting also to explore other solutions to the problem how to convert circular motion into straight-line motion. One of such examples is Cartwright mechanism that was used in weaving.

***Figure 15: Cartwright straight line mechanism***

We hope that material from our digital library can be used in schools not only in science or technology classes, but also as an example of using mathematical ideas in machine motion design. We are continuing to develop this library by adding stereolithographic files that allow 3-d printing. In Fig. 16 you can see a ratchet mechanism from Reuleaux collection and Fig. 17 is the working copy of the same mechanism printed in plastic.

***Figure 16: Ratchet mechanism***

***Figure 17: 3-d printed copy of the ratchet mechanism***

Cornell faculty in mechanical engineering, mathematics and architecture are using the models in the classroom to teach mathematical principles of mechanisms as well as machine design and drawing. Mathematical ideas from this collection have found its place in geometry textbook [3]. Over the past two years, students and teachers from area schools have visited the collection. We had as visitors aerospace engineers seeking ideas for microelectromechanical machines (MEMS), robotic machines, space satellite applications and biomechanical prostheses. The collection also has attracted scholars from Japan, Italy, Germany and Australia in the past year alone. We also have evaluation team working on this project and helping us better understand web design from users point of view [7].

The theme of the new Duffield Hall exhibit is the evolution of machine invention. The six displays feature mechanical models relating to the history of Cornell engineering, the machines of Leonardo da Vinci and Reuleaux, engines, kinematics and mathematics, clock mechanisms and modern applications of kinematic mechanisms. In addition to the 230 Reuleaux models, the collection contains 70 other rare mechanical models and measuring instruments, including 19th century slide rules and mechanical calculating devices. We are adding to our digital library Clark model collection that is in Boston Museum of Science and are working on further developing educational uses of this digital library for schools and colleges.

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