

The International Commission on Mathematical Instruction What? Why? For Whom?

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Mathematics is an international—even universal—discipline, and this aspect finds institutional expression every four years in the International Congress of Mathematicians (ICM), where a world community of mathematicians assembles to report progress on shared problems and with a common technical language. But these things, which we now take for granted, were not always so. The “international movement” in mathematics took hold only at the end of the nineteenth century, the first congress being the 1897 ICM in Zürich. The body that now provides the international infrastructure for mathematics—for example, sponsoring the ICM’s—is the International Mathematical Union (IMU),¹ an international nongovernmental and non-profit scientific organization with the purpose of promoting international cooperation in mathematics. The members of the IMU are not individuals but countries, and the adhering (membership) entity in each member country is typically its Academy of Sciences or a professional organization of mathematicians. The IMU is known to mathematicians mainly indirectly through the publicly visible expressions of its work, like the ICM’s and the awarding of the Fields Medals.

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¹<http://www.mathunion.org/>.

Our focus here is the International Commission on Mathematical Instruction (ICMI), the *mathematics education* counterpart to the IMU.

A Brief Bio of the ICMI

The International Commission on Mathematical Instruction² was founded at the fourth ICM held in Rome in 1908.³ It was initiated to support a then widespread interest among mathematicians in school education. The Rome Congress adopted a resolution, submitted on the initiative of the American mathematician, teacher-educator, and historian of mathematics David Eugene Smith (1860–1944), creating an international commission with the initial mandate of making “a comparative study on the methods and plans of teaching mathematics at secondary schools” (quoted in [5, p. 13]). The idea of such an international commission had in fact been formulated by Smith himself three years earlier in the newly established journal *L’Enseignement Mathématique* (L’EM) in his response to a survey proposed by the editors on the “conditions to be satisfied by a complete—theoretical and practical—teaching of mathematics in higher institutions” [2].

The birth of the ICMI was not modest. The great German mathematician Felix Klein (1849–1925), for whom mathematics education was a deep and career-long interest, became its founding

²Historical information about the ICMI can be found in the ICMI 75th-anniversary paper [3] by Howson as well as in Lehto’s recent book [5] on the history of the International Mathematical Union.

³So we are close to the first centenary of the ICMI, to be celebrated in Italy in 2008.

president, while the first secretary-general was Henri Fehr (1870–1954) from Switzerland, one of the co-founders of L'EM.⁴ The initial ICMI mandate, an international comparative study, ultimately became a massive six-year project producing 187 volumes containing 310 reports from 18 countries [5, p. 14]. Later ICMI presidents—thus far always research mathematicians—include such figures as Jacques Hadamard (from 1932 to the war), Marshall H. Stone (1959–62), André Lichnerowicz (1963–66), Hans Freudenthal (1967–70), Hassler Whitney (1979–82), and Jean-Pierre Kahane (1983–90), and Miguel de Guzmán (1991–98).⁵ So the interest and productive engagement of serious research mathematicians with school mathematics education, even at the international level, has a long and substantial, albeit uneven, history.

A small but significant place for mathematics education was reserved at the ICM's, in a section initially called "Teaching and History of Mathematics". It was in this section at the 1900 ICM in Paris that David Hilbert gave the talk "Mathematical problems" that shaped much of twentieth-century mathematics. As the history of mathematics later acquired a section of its own, the name changed to "Teaching and Popularization of Mathematics" and most recently to "Mathematics Education and Popularization of Mathematics", reflecting the broader nature of the field.

Over time, as the mission of general education expanded (more advanced knowledge, for more people), the needs and complexity of mathematics education grew as well, leading to the development in due course of corresponding communities of both practicing professionals and scholars. The small venue afforded by the one section of the ICM's became inadequate for the communication of problems and ideas in this expanded domain. This led ICMI president Hans Freudenthal to organize the first International Congress on Mathematical Education (ICME) in Lyon in 1969. These ICME's have since evolved into quadrennial congresses in years divisible by four,⁶ the next one to be ICME-10 in Copenhagen, July 4–11, 2004, where we expect some 3,500 participants, including a

significant number of mathematicians. (More on ICME-10 below.)

Connecting Mathematics and Mathematics Education

Mathematics education and *mathematics*, though obviously linked, are fundamentally different as domains of practice and scholarship. Their main historical intersection has been the induction and advanced mathematical preparation of mathematical researchers and scientists, a small but now growing fraction of the population served by school education, and this primarily at postsecondary levels. While most mathematicians teach, mathematics education treats teaching much more seriously as a professional practice, requiring dedicated training and certification. Theories of learning and of assessment play much more prominent roles, as does curriculum analysis. And these have become as well multidisciplinary domains of mathematics education research, using a variety of methods, many of them unfamiliar to most mathematicians.

So how are mathematics and mathematics education, as domains of knowledge and as communities of practice, now linked, and what could be the most natural and productive kinds of connections? The ICMI represents one historical, and still evolving, response to those questions at the international level. First of all, the ICMI, now formally constituted as a commission of the IMU (see the next section), is thus structurally tied to mathematics. In fact, nominally and in terms of governance, the ICMI is subordinate to mathematics. (Recall, for example, where the presidents of the ICMI have come from.) This is, on the one hand, a unique and potentially invaluable resource. Elsewhere in mathematics education—for example, in the fields of practice and in the institutional arrangements within universities—mathematics and mathematics education exist in different worlds that rarely communicate with each other and most often not well when they do. Yet history teaches us that there is a tradition of healthy interest and engagement of (some) mathematicians in mathematics education, and current experience indicates strongly that mathematicians have vital things to contribute, in multidisciplinary settings, to mathematics education, a potential not always sufficiently appreciated by mathematics educators. On the other hand, mathematicians sometimes lack a sufficient knowledge and/or appreciation of the complex nature of the problems in mathematics education, and they often tend to see issues of

⁴An initial aim of the "international journal" (to use its own description) *L'Enseignement Mathématique*, launched in 1899, was "to associate the world of teaching to the 'great movement of scientific solidarity' which was emerging at the end of the 19th century" [1, p. 11]. From the very beginning of the ICMI, *L'EM* was adopted as its official organ, which is still the case today. The other current channels of communication of the commission are the ICMI Bulletin, published twice a year, and the ICMI web-site, <http://www.mathunion.org/ICMI/>.

⁵It is with great sadness that we report the untimely passing of Miguel de Guzmán on April 14, 2004. Information about his life and work can be found at <http://www.xena.ad/1cf/fev2002/guzman.htm>.

⁶The successive ICME's, from ICME-2 in 1972 to ICME-9 in 2000, were held respectively in Exeter (UK), Karlsruhe (Germany), Berkeley (USA), Adelaide (Australia), Budapest (Hungary), Québec (Canada), Sevilla (Spain), and Tokyo/Makuhari (Japan). The ICMI has recently accepted the invitation received from Mexico to host the 11th ICME in 2008.



International Commission on Mathematical Instruction

mathematical integrity and rigor of the curriculum as the beginning and end of the story. (See the *Notices* article by Tony Ralston [6] for an insightful recent commentary on this cross-boundary behavior.) This kind of stance and disposition presents problems when it infects positions of authority and policy setting in mathematics education. In particular, this exposes a latent danger in the inherited governance arrangements of the ICMI, completely under IMU control. Many in the ICMI community argue for greater ICMI autonomy, yet still within the IMU environment.

ICMI vice president Bent Christiansen argued in 1982 that (see [5, p. 260]) the ICMI should *not* decide what are proper or relevant solutions to problems in mathematics education, but should provide a structure under which interaction and exchange of views could be facilitated. It should provide the type of leadership and structure responsive to the needs and interests of the growing mathematics education community and do so under the auspices of the IMU. He also voiced the frequently expressed need of a permanent secretariat for the ICMI.

Jean-Pierre Kahane, in his 1990 Farewell Message as president of the ICMI, gave the reasons for the ICMI being a commission of the IMU, in terms of

...the intimate link between mathematics and its teaching. In no other living science is the part of *mise en forme*, *transposition didactique*, so important at a research level. In no other science, however, is the distance between the taught and the new so large. In no other science has teaching and learning such social importance. In no other science is there such an old tradition of scientists committed to educational questions ([4, p. 6]; see also [5, p. 262]).

The Structure of the ICMI Today

Structurally the ICMI now exists as a member of the IMU family. After interruptions of activity around the two world wars, the ICMI was reconstituted in 1952, at a time when the international mathematical community was being reorganized, as an official commission of the International Mathematical Union.⁷ This still defines the formal position of the ICMI today. Thus, the Terms of Reference of the ICMI are established by the General Assembly of the IMU, which is also responsible for the election of the Executive Committee, the administrative leadership of the ICMI. Furthermore, the vast majority of the funding of the ICMI comes from the IMU. Once these election and budget matters are settled, the ICMI works with a large degree of autonomy.

As is the case for the IMU, members of the ICMI are not individuals, but countries—namely, those countries which are members of the IMU and other countries specifically coopted to the commission. There are currently eighty-one members of the ICMI, sixty-five of which are also members of the IMU. Each member of the ICMI appoints a representative and may create a subcommission for the ICMI. Such a subcommission serves the dual purpose of (a) providing an organized local (national) forum for dealing with issues of mathematics education and for exchange of information, and (b) offering a link between the local and international mathematics education communities. There are currently fourteen such subcommissions.

The ICMI's organizational outreach includes five permanent so-called Affiliated Study Groups, each

⁷ Through the IMU, the ICMI thus belongs to the International Council for Science (ICSU). This implies that the ICMI is to abide by ICSU statutes, one of which establishes the principle of nondiscrimination and free circulation of scientists. Lehto's book [5] vividly testifies to the importance of this rule in the life of both the IMU and the ICMI.

focusing on a specific field of interest and study in mathematics education consistent with the aims of the commission. The Affiliated Study Groups are neither appointed by the ICMI nor operate on behalf or under the control of the ICMI. They are thus independent of the ICMI for their work, also in terms of finances, but they collaborate with the ICMI on specific activities, such as the ICMI Studies or components of the program of the ICME's. They present reports on their activities to the General Assembly of the ICMI. The current ICMI Affiliated Study Groups, with their year of affiliation, are:

- HPM—*The International Study Group on the Relations between the History and Pedagogy of Mathematics* (1976),
- PME—*The International Group for the Psychology of Mathematics Education* (1976),
- IOWME—*The International Organization of Women and Mathematics Education* (1987),
- WFNMC—*The World Federation of National Mathematics Competitions* (1994), and
- ICTMA—*The International Study Group for Mathematical Modelling and Applications* (2003).

According to its Terms of Reference, the commission is “charged with the conduct of the activities of IMU, bearing on mathematical or scientific education and [takes] the initiative in inaugurating appropriate programs designed to further the sound development of mathematical education at all levels, and to secure public appreciation of its importance.” This is clearly reflected in the objectives and activities of the ICMI, which have considerably expanded over the years.

Activities of the ICMI

The ICMI's objective today could be globally described as offering researchers, practitioners, curriculum designers, decision makers, and others interested in mathematical education a forum for promoting reflection, collaboration, exchange and dissemination of ideas, and information on all aspects of the theory and practice of contemporary mathematical education, as seen from an international perspective. How does the commission achieve such aims? We now discuss some of its current major activities and programs.

The ICMI's primary responsibility is to plan for the ICME's, which entails choosing from among host country bids, appointing an international program committee to form the scientific program and select presenters, and overseeing progress of the congress preparations. But the practical and financial organization of an ICME is the independent responsibility of a local (national) organizing committee. This parallels to a large degree what the IMU does for the ICM's. However, the format of an ICME differs interestingly from that of an ICM in that it tends to be much more interactive, involv-

ing working groups and study groups that have structured meetings throughout the congress and that may well produce published reports for the congress proceedings.

ICME-10 in Copenhagen in July 2004 (see the next section) will also feature the first awarding of two medals in mathematics education research, recently inaugurated by the ICMI. The Felix Klein Medal for lifetime achievement will be awarded to Guy Brousseau from France. The Hans Freudenthal Medal for a major program of research will be awarded to Celia Hoyles of the UK. Information about these awards and citations of the work of the laureates can be found on the ICMI website, <http://www.mathunion.org/ICMI/>.⁸

A second major ICMI program is the series of ICMI Studies, a most successful set of activities launched in the mid-1980s. Each study focuses on a topic or issue of prominent current interest in mathematics education. Its International Program Committee (appointed by the ICMI) first drafts a “discussion document” that articulates the theme and purpose in great detail. This is widely disseminated to solicit papers from the field. From these submissions, invitations are issued to about eighty participants in an international conference, the results of which are synthesized into a research volume presenting a state-of-the-art expert report on the study theme. This process typically stretches over a period of about three years per study. These studies have acquired a growing importance and influence on the field. Here is a chronological list of past and current studies and their conference sites.

1. *The Influence of Computers and Informatics on Mathematics and Its Teaching* (Strasbourg, France, 1985)
2. *School Mathematics in the 1990s* (Kuwait, 1986)
3. *Mathematics as a Service Subject* (Udine, Italy, 1987)
4. *Mathematics and Cognition* (book prepared by PME, an Affiliated Study Group of ICMI)
5. *The Popularization of Mathematics* (Leeds, UK, 1989)
6. *Assessment in Mathematics Education* (Calonge, Spain, 1991)
7. *Gender and Mathematics Education* (Höör, Sweden, 1993)
8. *What Is Research in Mathematics Education and What Are Its Results?* (College Park, USA, 1994)
9. *Perspectives on the Teaching of Geometry for the 21st Century* (Catania, Italy, 1995)
10. *The Role of the History of Mathematics in the Teaching and Learning of Mathematics* (Luminy, France, 1998)

⁸ An announcement about the ICMI medals appears in “Mathematics People” in this issue of the Notices.

11. *The Teaching and Learning of Mathematics at University Level* (Singapore, 1998)
12. *The Future of the Teaching and Learning of Algebra* (Melbourne, Australia, 2001)
13. *Mathematics Education in Different Cultural Traditions: A Comparative Study of East-Asia and the West* (Hong Kong, 2002)
14. *Applications and Modelling in Mathematics Education* (Dortmund, Germany, February 2004)
15. *The Professional Education and Development of Teachers of Mathematics* (Águas de Lindóia, Brazil, May 2005)
16. *Challenging Mathematics in and beyond the Classroom* (Trondheim, Norway, June 2006)
17. *Technology in Mathematics Education* (2006)

The themes of these studies illustrate some of the many domains of mathematics education in which mathematicians, among other expert professionals, have crucial knowledge and expertise to contribute.

The study volumes for the first five studies were published by Cambridge University Press. Since Study 6, the study volumes appear in the New ICMI Study Series (NISS) published by Kluwer Academic Publishers⁹ under the general editorship of the president and secretary-general of the ICMI.

Another component of ICMI activities comprises the so-called ICMI Regional Conferences. Despite the international nature of its position and role, the ICMI from time to time lends its name to a variety of regional conferences on mathematics education, primarily in less affluent parts of the world. A number of ICMI Regional Conferences have thus been held over the years. These meetings are supported morally by the ICMI and sometimes with modest financial contributions as well. Recent and forthcoming ICMI Regional Conferences include:

- SEACME-8—*8th South East Asian Conference on Mathematical Education* (Quezon City, Philippines, 1999)
- *All-Russian Conference on Mathematical Education* (Dubna, Russia, 2000)
- ICMI-EARCOME-2—*Second ICMI East Asia Regional Conference on Mathematics Education* (Singapore, 2002)
- XI-IACME—*11th Inter-American Conference on Mathematics Education* (Blumenau, Brazil, 2003)
- EMF 2003—*Espace mathématique francophone 2003* (Tozeur, Tunisia, 2003)
- ICMI-EARCOME-3—*Third ICMI East Asia Regional Conference on Mathematics Education* (Shanghai, China, 2005)
- EMF 2006—*Espace mathématique francophone 2006* (Sherbrooke, Canada, 2006)

In addition to the above activities of a regular nature, the ICMI takes other initiatives on an ad hoc

⁹<http://www.wkap.nl/prod/s/NISS>.

basis. For instance, the ICMI is currently collaborating with UNESCO on an international exhibition on the theme “Why Mathematics?” aimed particularly at young people, their parents, and their teachers. This exhibition will be launched at the 4th European Congress of Mathematics in Stockholm in June 2004. It will then be shown at ICME-10 in Copenhagen and will later travel to various places.

A major aim of the commission is to support the development of mathematics education in less-affluent regions of the world. To this end, the ICMI initiated in the 1990s a Solidarity Program in Mathematics Education based on a twofold approach.

The first component of this program is the ICMI Solidarity Fund, established by the ICMI in 1992 at the suggestion of its president, Miguel de Guzmán. The overall objective of the Solidarity Fund is to increase, in a variety of ways, the commitment and involvement of mathematics educators around the world in order to help the progress of mathematics education in those parts of the world where there is a need for it that justifies international assistance and where the economic and socio-political contexts do not permit adequate and autonomous development. This initiative thus aims to foster solidarity in mathematics education between well-defined quarters in developed and less-developed countries. Particular emphasis is placed on projects that enable the activation of a self-sustainable infrastructure within mathematics education in the region, country, or province at issue. Central to this program of international assistance was the establishment of a fund to provide financial support for the approved projects. The Solidarity Fund is based on voluntary donations from individuals and organizations and is kept separate from the ICMI's general funds.

The second component of the ICMI Solidarity Program aims at having a balanced representation from all over the world among the presenters and the general participants in activities such as the ICMI Studies or the ICME's. In support of this goal, the ICMI has implemented, starting with ICME-8 in 1996, a general policy of forming for each ICME an ICME Solidarity Fund established by setting aside 10 percent of the registration fees in order to provide grants to congress delegates from nonaffluent countries. At each of the recent ICME's, some 100 to 150 participants from economically challenged regions of the world have thus been given financial support to facilitate their presence at the congress.

In the same spirit, efforts are made by the organizers of each ICMI Study to find financial resources so as to facilitate the participation in the Study Conference of a substantial delegation from nonaffluent countries.

An Invitation to ICME-10

The 10th International Congress on Mathematical Education, ICME-10, will be held in Copenhagen on the campus of the Technical University of Denmark on July 4–11, 2004. A distinctive flavor of ICME-10 is the fact that it is being hosted, not by a single country, but by the ensemble of the Nordic countries—Denmark, Finland, Iceland, Norway, and Sweden. The congress is expected to gather around 3,500 participants, including mathematicians with an interest in education, researchers in mathematics education, and teachers from all over the world. The International Program Committee, chaired by Mogens Niss¹⁰ (Denmark), proposes a structure combining the best from the ICME tradition with a number of innovative elements and features. The program, whose details can be found on the congress website,¹¹ includes the following events:

- 8 plenary activities, among which are Plenary Lectures by Hyman Bass (USA), Erno Lehtinen (Finland), Andreas Dress (Germany), and Ferdinando Arzarello (Italy); and reports from so-called survey teams that will present the state of the art with respect to themes such as reasoning, professional development of teachers, testing, or technology, with particular regard to identifying and characterizing important new knowledge, recent developments, new perspectives, and emergent issues;
- 80 regular lectures, covering a wide spectrum of topics, themes, and issues;
- 29 Topic Study Groups, some being organized according to educational levels, others according to content-related issues, and the rest to overarching perspectives and meta-issues;
- 24 Discussion Groups focussing on the examination and discussion of issues that can be dealt with in different ways depending on experiences, values, norms, and judgments;
- a thematic afternoon with 5 parallel miniconferences: *Teachers of mathematics*, *Mathematics education in society and culture*, *Mathematics and mathematics education*, *Technology in mathematics education*, and *Perspectives on research in mathematics education from other disciplines*;
- special sessions of the 5 ICMI Affiliated Study Groups and reports on the 3 most recent ICMI Studies;
- various sets of activities, such as workshops, Sharing Experiences Groups, poster exhibitions, paper presentations, or round-table sessions;
- national presentations from the Nordic countries (Denmark, Finland, Iceland, Norway, and Sweden), Korea, Mexico, Romania, and Russia.

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¹¹<http://www.ICME-10.dk/>.

A congress such as ICME-10 provides a unique opportunity to learn about recent developments in mathematics education around the world and to be introduced to innovations and recent research on the teaching and learning of mathematics at all levels, from primary to tertiary education. The scientific program aims to provide food for thought and inspiration for practice to all, from the established mathematics educator to the novice in the field, and to all with an interest in mathematics education. It is structured so as to allow plenty of choice while encouraging exchange and contacts between participants.

In a context where the debate over the state of mathematics education at all levels, from primary school to graduate school, is becoming more intense and vigorous than ever, ICME-10 provides unparalleled access to expert knowledge in the field. ICME-10 can surely play a significant role in facilitating the exchange of ideas and experiments within and between the mathematics and mathematics education communities and contribute to the improvement of mathematics education all around the world.

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