Understanding "hidden rules": the challenge of becoming a competent member of a mathematics classroom

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INTRODUCTION

In his invited address to the Special Interest Group in Research in Mathematics Education at the annual meeting of the NCTM in 1979, Heinrich Bauersfeld spoke about "hidden dimensions in the so-called reality of a mathematics classroom" and argued for researching these dimensions. While suggesting the study of the interactive constitution of shared meanings in classrooms, he also reminded the audience of considering the impact of the institutional settings. Institutions "constitute norms and roles", "develop rituals in actions and in meanings", "tend to seclusion and self-sufficiency" and "even produce their own content – in this case, school mathematics" (Bauersfeld, 1980, pp. 35-36). Bauersfeld suggested that ethnomethodology and linguistics provide promising theoretical bases for a research agenda that addresses the hidden dimensions of mathematics classrooms.

Whether what is going on "below the surface" in mathematics classrooms remains not only hidden to the students and teachers, but also to the researcher, is a matter of methodology and theorizing. Since Bauersfeld gave his address, many researchers in mathematics education have come to investigate what he indicated by "the hidden dimensions in the so-called reality of a mathematics classroom" in order to understand how these afford or constrain students' access to mathematical knowledge. The most prominent theories employed in empirical classroom research to achieve this goal include Symbolic Interactionism and Phenomenology, in particular Ethnomethodology, as well as theories that are concerned with the social reproduction through schooling, such as those of Bourdieu and Bernstein. But also some theorizing or compilations of other theories that emerged from within mathematics education as a research domain addresses the problématique.

CONCEPTUALISING "HIDDEN RULES"

The following episode from a mathematics classroom illustrates some dimensions of what the title intends to indicate by "hidden rules". Meyer (2010) discusses some episodes from a 4th grade classroom in Germany in which the teacher intends to introduce the notions "parallel", "perpendicular" and "right angle". The terms are written on the board. After asking the students to freely associate what comes to their minds, a reproduction of a painting by Mondrian is shown to the students.

> „Teacher: Why do I fix such a picture on the blackboard? And why are these concepts written down on the blackboard? I have a reason to do so. Jonathan, it is your turn.
> Jonathan: Because the painter has done everything in parallel, perpendicular and in right angles.
> Teacher: You are right. You seem to know what parallel, perpendicular and right angle means. Maybe you can show it to us on the picture.
> Jonathan: Perpendicular is this here (points first at a vertical, afterwards at a horizontal line). Parallel is this here (points at two vertical lines). A right angle is this (pursues two lines he former would have called perpendicular)."
> (Meyer, 2010, p. 909)

Meyer, by drawing on Wittgenstein's notion of language-games, discusses the scene as an instance of establishing the "exemplaric use" of words in this classroom. He also points out that Jonathan must have been participating in practices of using the words "parallel", "perpendicular" and "right angle" in a similar language-game outside this classroom.

However, the episode shows that Jonathan had to know more than how to engage in the language game of ostensive definitions that employ visual recognition. For producing his positively sanctioned answer, Jonathan also had to understand the question as a prompt to associate the notions written on the board with the configuration of lines in the painting. Alternative replies that might have been produced without understanding the actual illocutionary act performed by the teacher's question, such as "because you like the painting", most likely would have been taken as an expression of sarcasm by the participants. In addition, Jonathan had to recognise that the teaching here is organised as a series of related questions to be answered or discussed by the students, and to have access to the criteria for producing an appropriate contribution to a description of a piece of art in a geometry lesson, in contrast, for example, to a discussion of the style of the painting in an arts lesson.

In this episode, different dimensions of "hidden rules" become visible. The (emergent) rules for using and producing mathematical signs and for a legitimate way of presenting an externalisation of one's thinking according to these rules (orally or in a written form), the rules of the pedagogical principle adopted by the teacher that account for the establishment of routines in communication, the rules that constitute the specificity of the school mathematical practice and its discourse in relation to other practices and their discourses, as well as the norms for favoured behaviour, aspirations and attitudes. As all these rules regulate how students relate to and gain access to different forms of mathematical knowledge, the challenge for the students is to acquire knowledge of these rules in order to develop the skills that are necessary for successful participation. This opens up the question of whether al students have equal access to these rules. There might be hidden principles in operation that account for the stratification of achievement because not all students gain equal access to the knowledge code.

At this point, a remark on terminology seems advisable. Because of lack of alternatives, in the heading the term *rule* is used as an umbrella term, referring to *norms, specific rules, routines* and *principles*. The term *norm* often refers to established standards to be achieved, sometimes also to typical patterns found in social actions. Typical patterns of actions that are carried out repetitiously and often are followed unconsciously might be classified as *routines* (or *rituals*). A *specific rule* combines a norm that consists of specific criteria with a regulation for achieving them. The term *principle* indicates an underlying invisible mechanism. According to this differentiation of meanings, specific rules, norms, and principles differ in stability, visibility, accessibility and relations to a wider system.

Specific rules

The notion of a *specific rule* is used here to suggest that it refers to norms that condense a set of specific criteria for an action in a regulation for achieving them. Many of such specific rules in a mathematics classroom are about the behaviour and the social organisation of the work. The rules might be unspoken, but if asked, many students would be able to express them: "When we work individually, the we are actually allowed to discuss with the students sitting next to us." "We can ask questions, when we get stuck in a task and the teacher will then come to our desk and help us." "When the teacher writes something on the board, we have to copy it into our notebooks." "We always have to write down the answer to a word problem as a full sentence." "The result of a calculation has to be double-underlined." "When the teacher says 'tell me more about this', she wants us to show how we calculated it." "She wants us to just work on the warm-up and get the answers for it, and then later she asks us for the answers so that we can correct ourselves." The students are more or less conscious of such rules. Consciousness opens up a space for tactical behaviour. And only if the students are consciously aware of the rules, they can intentionally not comply, which can then be interpreted as an act of resistance.

To the chagrin of many mathematics educators, teachers often introduce explicit specific rules for solving certain types of mathematical problems. In a comparative study of six year-8 mathematics classrooms[1], two of which were from Germany, Hong Kong and the United States respectively, Jablonka (2004) found examples of explicit guiding manuals for solving tasks in classrooms of all three countries. These included manuals for tackling word

[1] The study was part of the Learner's Perspective Study, see extranet.edfac.unimelb.edu.au/DSME/lps/)

problems and careful consideration of single steps conceptualised as rules in algebraic transformations. In a Hong Kong classroom, for example, the teacher introduced a six-step procedure for setting up equations in two variables (called "unknowns") in order to solve word problems: (1) *Examine*, (2) *Let* (short for „let *x* be, let *y* be…"), (3) *Form* (two equations), (4) *Solve*, (5) *Check*, (6) *Answer*. In line with others, Jablonka (2004) also found a preference on the side of the students for step-by-step manuals for solving tasks. Many students referred to a set of explicit and detailed rules as a good "explanation" by their teachers.

If criteria for actions are transformed into regulations for achieving them, the criteria remain implicit, and validation of the outcome can only be achieved through checking the correctness of the procedure, but not in relation to the criteria. The students will not be able to check the validity of their solutions in relation to the problems to be solved, and not get used to invent ways of solving unfamiliar problems. Consequently, such a focus on teaching explicit rules has been an ongoing concern of mathematics educators. The alternative typically consists in presenting a sequence of problems so that the students themselves can construct a general underlying meaning structure. As Ernest (2006, p. 75) points out, there remains an unresolvable tension between leaving the general principle implicit or rendering it explicit: "Thus the paradox is that general understanding is achieved through concrete particulars, and specific responses only may result from general statements."

However, there is a price to be paid for leaving the work of constructing more general mathematical meanings to the students in inquiry based mathematics classrooms. For example, Theule Lubienski's (2000) study in what has become called a reform mathematics classroom, shows that students did not equally make use of the open whole-class discussions. While high-socioeconomic status students were able to recognise the importance of looking for generalisations, lower-socioeconomic status students focused more on giving correct answers to specific, contextualized problems and could not fully appreciate the presentation of a diversity of ideas but preferred more teacher direction. Jablonka (2004) found that many of the lower-achieving students felt lost as soon as open-ended tasks were introduced that allow for different solution strategies. Teese (2000, p. 171) reports from a reform project in Victoria, Australia, in which an inquiry-based curriculum has been followed. The approach turned out to be of disadvantage for working class girls. This group was more successful in the traditional setting. Dowling (2009) shows how an investigative approach to school mathematics introduces new skills and "tricks". What makes such skills or tricks meaningful for the construction of new mathematical knowledge can only tacitly be decided on the grounds of previously acquired mathematical knowledge. Similar concerns can be raised about approaches that favour teaching mathematics through mathematical modelling.

Norms of classroom practice as (emerging) conventions

Emerging norms embody expectations and values that are supposed to be shared by the group about what is an appropriate contribution to the practice. These norms can be reconstructed from an observers' point of view by the fact that most participants show some signs of having adopted the expected actions at some stage. The reconstruction resembles an ethnographer's re-construction of the "folk-ways". Such an interpretation of classroom practice will be a hermeneutic, immanent one. But it is not done by the participants who are involved, except, perhaps, in the case of a breakdown of the smooth flow of co-ordinated actions. The "socio-mathematical" and "social" norms (e.g. Yackel & Cobb, 1996), the "didactical contract" (Brousseau, 1980), and some of the "meta-discursive rules" (e.g. Sfard, 2001) refer to these types of norms.

The focus in studies of classroom practice is often on the changing character of the norms when the construction of new mathematical knowledge is at issue. Voigt (1984) studies regularities in mathematics classroom interaction in relation to the learning behaviour of the students. He assumes that teacher and students are in the possession of unconscious practices or routines (Schütz & Luckmann, 1975) that help them to structure the process of constituting knowledge that eventually counts as shared knowledge. The notion of *routine* here refers to the fact that these interaction patterns are unconsciously accomplished, have the function of reducing the complexity of the situation, and yield a harmonising effect. Voigt (1984) analyses variations of a common whole class pattern of interaction in German mathematics classrooms that is called the *fragend-entwicklendes Unterrichts-gespräch* [questioning-developing instructional talk]. There are similar terms in other European countries, as for example the *onderwijs leergesprek* [classroom teaching talk] in Dutch. In classrooms from the U.S.A., "guided development" resembles a similar, perhaps more open form of such a pattern. Successful participation in this activity does not imply that all students share the mathematical meanings the teacher intended to constitute. The students might only have developed the competencies of how to participate in the interactive production of knowledge that is institutionalised. The pattern has been criticised as it affords acting according to the teacher's expectations. The students might spend much effort in finding out the implicit rules of the "didactical contract", which is constituted through mutual expectations and interpretations of "specific habits" of the teacher by the students and vice versa (Brousseau 1980, p. 180). This description of the didactical contract is reminiscent of the description of interpretive procedures described by ethnomethodology (e.g. Voigt, 1984, p. 23 ff.).

Voigt (1984, p. 22) observes that the functioning of the routines for the interactive construction of new knowledge is apparently contradictory. As there is no shared frame of reference from the outset, the teacher's initial question or task is necessarily ambivalent. But the task is reflexively bounded to its solution: Only retrospectively the official solution reduces the ambivalence of the question. The institutionalised solution constitutes the meaning of the task of which it is a consequence. Voigt (1984, p. 56) gives an example of classroom interaction, in which the routine is disturbed. The teacher asks the students to articulate "whether they can already notice something" [a pattern in the numbers written on the board]. The obligation is to bring about constructive contributions. In the example, a student complains: "What am I supposed to notice there?" The teacher replies: "What you are supposed to notice, this you have to know yourself. Björn [another student] can you notice anything?" The teacher evaluates the student's question as a violation of the obligation to try to answer his question that has to be assumed to make sense and be of (didactical) value.

That initial question necessarily has to be ambivalent in order to make the construction of new knowledge possible. The "funnel pattern" observed by Bauersfeld (see, e.g., Cobb & Bauersfeld, 1995) is a routine for narrowing down the scope of possible responses, without ever revealing what exactly the criteria for a valid contribution are. The pattern can be seen as the interactional manifestation of what Ernest (2006), from a semiotic perspective, refers to as the general-sepecific paradoxon (see above, the section on specific rules). Not all students are equally able to acquire and interpret the emerging expectations of what an appropriate contribution consists of.

Hidden principles related to a wider social context

The teachers and the students in a classroom are not free to redefine the practice of school mathematics. There are principles in operation that guarantee continuity of classroom practices. The teacher has an obligation to deliver the intended curriculum and reach a result that is defined by curriculum documents and assessment practices. Teaching is the mediation of the institutional culture by local personnel. Patterns of classroom interaction are functional in terms of the goals of the institution and are not accomplished at the initiative of the participants in a single classroom. One of these goals is channelling different groups of students into different career pipelines.

For analysing classrooms in relation to the institutional context, a layer of interpretation has to be introduced that goes beyond the reconstruction of the participants' interpretations (the individual students' learning) and beyond the reconstruction of classroom norms. The participants' ways of acting is then interpreted by using information and theories, which the participants (usually) are not aware of. This is to reveal the social function of what happens in classrooms caused by factors to which the students and teachers have no access. It is to re-construct those principles that function in covert ways and serve the interest of power in the social system, independently of the actors' intentions. Conceptualisation and investigation of these principles draws on structuralist and critical theories. This section outlines some issues and outcomes of research dealing with principles that account for unequal attainment.

RECONTEXTUALISATION, DISRUPTIONS AND DISCURSIVE GAPS

It has been argued from different perspectives that school mathematics differs fundamentally from other types of mathematics, especially from the practice of researching mathematicians. School mathematics has a distinct epistemological character, its own systems of symbolising and a knowledge structure that is different from other mathematical practices. The culture of the mathematics classroom brings about a specific type of mathematical knowledge and mathematical language (Steinbring, 1998). Anna Sfard (1998) proposes that mathematicians and mathematics educators' views of mathematical knowledge might even be incommensurable. The disparity between different institutionalised mathematical practices and the forms of knowledge developed in these practices can be seen as the "raison d'être" of the French "Antropological Theory of Didactics".

Mathematics classrooms belong to a special type of practice, that is, to pedagogical practices. In this classrooms are very different from other practices, in which mathematics is used and developed. Pedagogic discourse is achieved by a principle of recontextualising other discourses. Recontextualisation (e.g. Bernstein, 2000; Dowling, 2009) points to the transformation of discourses that are moved from one social context to another. The process brings about the subordination of one discourse under the principles of the other. Bernstein (2000, p. 33) sees pedagogic discourse as constructed by a recontextualising principle which selectively appropriates, relocates, refocuses and relates other discourses to constitute its own order. Hence, pedagogic discourse can never be identified with any of the discourses it has recontextualised. School mathematics commonly not only recontextualises academic mathematics, but also outside-school practices. As school mathematical discourse is not static, but changes according to some progression in the curriculum, learning in a mathematics classroom can be described as moving through a range of practices and their constituting discourses, in which students have to successfully participate. Many students get lost on the way.

PROBLEMS WITH "INTERMEDIARY DOMAINS"

A common strategy to overcome the discursive gap between everyday discourse that has been described as exhibiting a "horizontal knowledge structure", and school mathematical discourse that resembles a "vertical knowledge structure" (e.g. Bernstein 2000), is the construction of intermediary domains:

> "As Anna Sfard shows us, in discussing the limits of mathematical discourse, the differences in the 'meta-discursive' rules between everyday discourse and mathematical discourse require us to develop a well-defined intermediary between the two."
>
> (Umland & Hersh, 2006, p.9).

Dowling (e.g. 2009) has described these intermediary domains as a collection of everyday objects and events that are recontextualised from the perspective of mathematics. This collection constitutes the public domain of school mathematics. This domain only becomes "well-defined" through a process of institutionalisation.

A recontextualisation brings about a new focus and a change of perspective. There are certain aspects to be sought after and others have to be dismissed and a decision is made about what is considered significant and what is accidental. New meanings and new relationships between meanings are established and at the same time new forms of expressions are introduced as well as new rules for elaborating their internal coherence. These changes in focus and signification (in "socio-mathematical norms") are rarely made explicit, except, perhaps, in the case of a breakdown of the smooth flow normally guaranteed through the interactional routine.

The following example, where the rolling of a dice is involved, shows the difficulty of the transition from everyday to mathematical meanings.

T: And if I said now roll a number smaller than one?
S: … won't work!
T: But this is also an event. Indeed, as you have already said correctly, this event…
S: … won't work! … Won't work!
T: Yes. How would we now attach an adjective to this?
S: … certain …
S: … the uncertain event.
T: The uncertain? Let us just call it the impossible event. And now my question. What subset is that actually, if I speak about the impossible event?
S: That won't work at all!

(Transcript translated from Steinbring, 1998, p. 164)

The task for the students is to see the activity of rolling a dice from the perspective of probability theory using a set-theoretic notation. As they recognise rolling dice from playing games, they interpret the teacher's questions in terms of the discourse belonging to this everyday domain, and there is of course no expression for rolling a number smaller than one. However, in the next turn, the teacher uses specialised language, such as *event* and *subset*, while only the latter might be recognised as such. This is understood at least by one student as a hint that this is not about playing games, but about rolling a dice from the perspective of school mathematics. When subordinating one practice (rolling dice) to the principles of another (school mathematics) it is always ambiguous to what extend the subordinated practice remains relevant. And this issue is even more complicated if the principles of the other practice are not completely known to the recontextualisers, that is, to the students (see Gellert & Jablonka, 2009, for further discussion).

Empirical evidence suggests that the institutionalisation of segments from everyday discourse within school mathematical discourse has a tendency to allocate the everyday insertions to marginalised groups (see, for example, Boaler, 1994; Cooper and Dunne, 1999; Dowling, 1998). The recontextualisation of domestic practices in school mathematics serves as means of stratification of achievement.

EPISTEMOLOGICAL DISRUPTION

The discursive gaps are not restricted to the problem with the "intermediary domain". In the course of a year-8 lesson about algebra in a Hong Kong classroom from the study quoted in the previous section (Jablonka, 2004), a disruption of meaning of "exact solution" is visible. A student suggests using a ruler for measuring the coordinates of the point of intersection in a Cartesian graph in order to get an "exact answer" of a system of linear equations. He learns that this is "not very accurate".

T: Okay. Continue with your work...everybody. It's difficult for you to look for the answer in question four...very difficult...very difficult.
T: What shall I do if I want to find the exact answer?
S: Use a ruler.
T: Huh? I want a very...very accurate answer.
S: Method of substitution.
T: Yes. Method of substitution...or?
S: Method of elimination.
T: Yes. Good. I'm going to look for the lazy bones that have done nothing. [Teacher walks around]

What is the meaning of "exact answer"? It is obvious that the student's suggestion was not satisfactory because the teacher repeats his initially ambiguous question in a slightly different version. The students might conclude that there is a seamless transition from accuracy of measurement to mathematical exactness, but it is in fact an epistemological difference, a difference in the quality of how the knowledge is warranted.

A discussion of how, what here has been called an "epistemological disruption", is linked to the students' background is provided by Gellert (2008). The analysis contrasts an interactionist with a structuralist analysis of an episode from a classroom.

HOW TO GUESS THE ESSENTIAL THING: RECOGNITION AND REALISATION RULES

The following example may serve as an illustration that the learner must know both, what Bernstein (e.g. 2000) calls *recognition rules* and *realisation rules*. There is a little piece of text. It is a quote from a book:

> "They kept on running, even though they were tired.
> At eight o'clock we begin studying.
> They will soon stop working.
> Usually Anita gets her cleaning done on Friday."
> The original language version (in Swedish):
> "De fortsatte springa fast de var trötta.
> Vi börjar studera klockan åtta.
> De slutar arbeta om en stund.
> Anita brukar städa på fredagarna."

What is this text about? Is there any relationship between these statements? Is there a storyline? Is this text coherent? What is the principle one has to know in order to construct a similar text?

The text is from a language course in Swedish for second language learners. The sentences are grouped together only for meta-textual reasons and there is no other relationship between the meanings. Hence the text hardly makes any sense in terms of everyday discourse. Discovering the meta-textual similarity is hard because in everyday contexts when using language, even if one is very competent, there is no need to be consciously aware of a distinction between meta-textual features and meaning. Knowing the context (a language course for foreigners) is necessary, but not sufficient. Command of the recognition rule is important for being able to locate classroom discourse, that is, to distinguish the specificity of this context. One has to pay attention to different things and one is positioned differently in relation to the others when one participates in a language course. But recognizing the specificity of the school discourse is not sufficient for successful participation. In addition one has to be able to utter ones thoughts in an appropriate way. Command of the realisation rule is important for the production of a legitimate contribution. Without command of the recognition rule, the problem is that one does not even know what it is that one does not understand. Without the realisation rule, one cannot participate. Recognition is a necessary condition for production.

Bernstein (e.g. 2000) deconstructs "invisible pedagogy" because of its differential effect stemming from the implicitness of the recontextualisation principle, which makes invisible the classificatory principle of the knowledge to be acquired and students do not have equal access to the recognition and realisation rules.

RESUMÉE

Not all the rules operating in mathematics classrooms are equally accessible to all students. Understanding or non-understanding shapes the control over participation and eventually determines who is included, excluded or marginalised. Teachers differ in the ways in which they provide access for the students to the organising principles of the discourse in ways that some practices are of advantage or disadvantage for distinct groups of students. In an ongoing study that involves classrooms from Canada, Germany and Sweden, the researchers collaborating in the project are concerned with the emergence of disparity in achievement in mathematics classrooms[2]. The project investigates the emergence of disparities from a theoretical perspective that examines their social construction in the context of the practices of the mathematics classroom while taking into account factors that might lead to the systematic exclusion of some students and to the success of others. The project seeks to identify and describe discursive and interactional mechanisms that can explain if and how structural elements can be found in classroom interactions. Hence, the questions asked include:

- How do teachers actually introduce students to the to the organising principles of the discourse? Are there distinct groups of

[2] See http://www.acadiau.ca/~cknippin/sd/index.html

students who benefit from these introductions? Who could benefit if this practice were different?
- At which moment in the course of a teaching unit or of a school year, on which occasion, do teachers provide an insight into the criteria along which the stratification of attainment within the mathematics classroom is achieved – if they do at all?
- What can the students articulate about the criteria?

As to the practice of teaching, describing the subtleties of the process might help to be more aware of it.

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