DG 22: Current problems and challenges in upper secondary mathematics education

Team Chairs: Neil Eddy (South Africa), neddy@wbhs.org.za
Denis Tanguay (Canada), tanguay.denis@uqam.ca
Team Member: Gwen Zimmermann (USA), gzimmermann@d125.org

This diverse group of secondary classroom teachers, mathematics educators, and mathematicians from several countries discussed a variety of topics. While group participants brought many themes forward, both accepted papers and their respective presentations identified additional challenges to consider. What follows is a brief summary of the rich and varied discussions of DG 22.

1. Curriculum reforms

Wang Linquan, PR China, presented a paper highlighting recent curricular changes in China at secondary level. He outlined the mathematics curricular structure and identified key mathematical concepts within the curriculum, noting that these recent changes create challenges for both teachers and students. Some of the mathematical content is beyond the current mathematics knowledge of teachers and little time or resources are available for professional development. Students are likely to be challenged, as many of the new topics will never have been seen before. Furthermore, students will need to learn topics quickly since no time will be available for teachers to slow down their instruction.

With the discussion that followed the presentation, we witnessed a general agreement about problems caused by curricular reforms, which have the unfortunate tendency to be alike from one country to the other. In the worst case, these problems are as follows.

- Teachers' lack of preparation with respect to new pedagogical trends and to changes in the content organisation. Teachers are seldom consulted, let-alone involved.
- New textbooks or schoolbooks written and edited in a hurry.
- Poor planning and coordination in organising content, 'holes' in the knowledge needed for further schooling, in-between student cohorts being tossed around by the reform.
- Difficulty for the teachers to adjust assessment to the new curriculum.

2. Technology

In the teaching and learning of mathematics, a variety of technological tools are available both to teachers and students. The financial constraints in some countries are a concern related to equity, as well as being problematic in their own way, and may exacerbate the rich/poor divide across the world. Much of the group's discussion focused on meaningful use of technology. Technology should not become a crutch for a student, but rather support the learning of mathematics so that students use technology to enhance learning and develop in their ability to reason mathematically. Giving examples, some participants argued that technology has the ability to open doors to problems that were previously closed to students. Others expressed a concern about technology driving mathematics away from deductive reasoning, by establishing new forms of empiricism based on purely computational validation. The attitudes towards technology vary from one country to another. Technology is banned from assessment in some countries, while others assess mathematical skills in separated calculator and non-calculator examinations. The discussion concluded with the agreement that the use of technology in the classroom should be careful, embedded in relevant and significant tasks and that it calls for adequate teacher training and the availability of rich pedagogical resources. Some web resources are:

www.geogebra.org/cms/	www.math.uqam.ca/APTE/	www.acdca.ac.at
www.cas-time.com/	www.lonklab.ac.uk/came/	http://kutzler.com

mathforum.org/	www.mathsnet.net	http://nlvm.usu.edu/en/nav/vLibrary.html
www.ed.psu.edu/casim/	nctm.org/	www.edfac.unimelb.edu.au/dsme/cas-cat
mathedleadership.org/		

New technologies are constantly emerging and often heavily marketed by profit-focused corporations. Decisions regarding the use of technology should be based on the ability of the teacher to use the technology to support student learning of mathematics—not on governmental or corporate initiatives.

3. Post-secondary education vs vocational training

Should secondary mathematics curricula distinguish between students bound for post-secondary education versus vocational training? After discussion questioning whether the level of content and rigour should be the same regardless of a student's future goals, the group focused on the question of whether we should "differentiate instruction or differentiate curriculum." Regardless of the anticipated educational track, all students should leave the classroom able to use the mathematics they learn. What is important is that the curriculum is coherent and designed to anticipate either educational goal. In the classroom, teachers need to be knowledgeable about content and pedagogy and flexible in their approaches to best meet the needs of their students. Finally, teachers need to be able to deliver the curriculum through rich tasks containing meaningful mathematics. There was a general consensus that mathematics courses must enhance reasoning and deeper understanding, even in more technical and vocational programmes, where the tendancy would be to favour procedural and utilitarian learning.

Universities need some of the following characteristics in their entry-level students: the concept of proof, motivation, a degree of fluency, confidence to deal with the complexities of the subject, and a stable curriculum experience. The skill of learning was considered of greater importance than specific mathematical skills. A concern was expressed that university programmes may tend to cover too much content. A student well-formed in how to learn will always be able to fill in the gap left by a less loaded curriculum, so that it appears to a majority among the participants that it is more a matter of 'how to learn' than 'what to learn'.

4. Assessment

High stakes assessment is typical. However, not all mathematics or mathematical reasoning is assessed. Assessing processes, such as problem-solving, is time consuming and often too difficult to be done on a large scale. In South China, teachers are also expected to assess student attitudes and dispositions, although not specifically about learning mathematics.

A brief discussion about "grade inflation" occurred. Grade inflation can come from sources other than the teacher due to pressure for high achievement. Some practices exist to reward high marks; for example, in Georgia, USA, students can receive free tertiary education based on their grades. The consensus was to deplore mathematics being used as a general assessment tool as discriminatory and selective, cf. Latin at the beginning of the twentieth century (see §6).

5. Mathematically Rich Activity

Denis Tanguay reported on a study done jointly with Denise Grenier (Université Joseph Fourier, Grenoble). Transitions from high school to the final years of high school and to the first year of university have been the subject of numerous studies in mathematics education. One of the most acute problems linked with transitional difficulties is the increasing use of formalism, symbolism and proof in post-secondary mathematics. Researchers (for example, Dorier & al., 1997 or Sierpinska & al., 1999) refer to this problem as 'the obstacle of formalism', pertaining to the problem of meaning, which teaching should bring to the classroom mathematical experience. As regards proof in particular, according to Grenier and Tanguay, it is a matter of resituating it within a process where it acquires full meaning, a process as complete as possible, and similar to the practice of the true mathematician-researcher. Their didactical assumption is that students to be regularly placed in the situation of experimenting, defining, modelling, formulating conjecturing and proving, with formal proof then appearing as a requirement in establishing the truth of the proposed conjectures.

The link between definition and proof has been examined in Tanguay (2007), where the author emphasises that learning formal proof goes through a necessary refocusing, from the truth of the contents to the validity of deductive sequences: the student must understand that it is no longer a matter of his producing true statements but rather valid steps of reasoning. This process involves a crucial shift from the pragmatic to the theoretical (Balacheff, 1987), where definitions have a fundamental role to play.

For an example of what Grenier and Tanguay evaluate as ‘a mathematically rich activity for upper secondary maths classes’, they proposed the Plato’s five polyhedra situation in Solid Geometry. For an experimentation based on this situation, the interested reader may consult Tanguay & Grenier (2009). For other so-called Situations de recherche pour la classe (SiRC), one may consult <http://mathsamodeler.ujf-grenoble.fr/>

Following his presentation, Denis Tanguay invited the group to share their insights on what constitutes a ‘rich mathematical task’. Participants agreed that characteristics of a rich task should include being meaningful and realistic (although not necessarily ‘real world’) and designed to help raise students’ cognitive levels, as well as increase their knowledge of mathematics content. Thus, the mathematics to be learned contributes to the richness of a task. A rich task will allow multiple ways to approach various answers, will encourage exploration by students, and will often be open-ended in its solution. When designing and using such tasks, teachers need to pay attention to framing and scaffolding the problem.

One challenge is the time needed to create such tasks. Teachers also need to have strong content knowledge as they need to be able to draw out the meaningful mathematics from the task. Once again, we are driven towards the crucial issues of (pre- or in-service) teacher training and the availability of pedagogical resources.

6. Students’ Attitudes

Most participants agreed that lack of student motivation was a concern. Over-emphasis on high-stakes assessments can lead to a negative perception of any material not directly beneficial to these tests. The teaching of mathematics needs to be engaging and meaningful to students. The use of mathematically rich tasks is one way to affect student motivation. Some teachers will attempt to positively affect a student’s negative attitude by seeking ways to make mathematics learning more accessible through the use of positive reinforcement. Mathematics has long been associated with intelligence leading some to believe that the ability to learn mathematics is predetermined. The attitude needs to be changed.

7. Teacher Time Constraints and Professional Development

Some countries and some schools build in time for professional development, such as for in-service training and on-line courses. The time available for professional development for teachers is often severely limited, however, and expensive to finance. For instance in the UK, it is normal to incur a cost of substitution cover when a teacher absents him/herself to benefit from in-service training, to provide some for others or to do work on question paper committees for public examinations. Other countries and other schools do not have schemes at all to provide for adequate time for teachers to collaborate, research and extend their skills.

References

Balacheff, N. (1987) Processus de preuve et situations de validation. *Educational Studies in Mathematics,* vol. 18, n°2, 147-176.

Dorier, J.-L., Harel, G., Hillel, J., Rogalski, M., Robinet, J., Robert, A., Sierpinska, A. (1997). *L'enseignement de l'algèbre linéaire en question.* Coord. par J.-L. Dorier. La Pensée Sauvage. Grenoble, France.

Sierpinska, A., Dreyfus, T. et Hillel, J. (1999). Evaluation of a Teaching Design in Linear Algebra : the Case of Linear Transformations. *Recherches en didactiques des mathématiques*, Vol. 19, n°1, 7-40.

Tanguay, D. & Grenier, D. (2009) A classroom situation confronting experimentation and proof in solid geometry. To appear in *ICMI Study 19 : Proof and proving in mathematics education*. National Taiwan Normal University.

Tanguay, D. (2007) Learning Proof : from Truth towards Validity. *Proceedings of the Xth Conference on Research in Undergraduate Mathematics Education (RUME)*, San Diego State University, San Diego, California. 15 pages, http://cresmet.asu.edu/crume2007/eproc.html