Discussion Group 27: How is technology challenging us to re-think the fundamentals of mathematics education?

Team Chairs: Lulu Healy, Bandeirante University of São Paulo, lulu@pq.cnpq.br
Lynda Ball, The University of Melbourne, lball@unimelb.edu.au
Ana Isabel Sacristán, DME-Cinvestav, asacrist@cinvestav.mx

Introduction

Although technology has been a key aspect of mathematics education for several decades, its role and use continue to be subject to great interest as well as considerable controversy and debate amongst mathematics educators and mathematicians. Undeniably, the introduction of technology into mathematics classrooms has the potential to motivate transformations of both educational processes associated with the teaching and learning of mathematics as well as the mathematics itself. But are such transformations realised in practice?

We can certainly point to potential for change. As the 20th century has given way to the 21st, the rapid development of digital technologies through which to practice mathematics continues, with seemingly no slowing in pace. Technological advances have made available an ever-increasing array of dynamic means for representing mathematical objects in addition to new communication infrastructures, through new forms of synchronous and asynchronous connectivity. However, although the digital technologies available for use in mathematics classrooms, laboratories and lecture halls are constantly evolving, the rate of change in educational institutions proceeds according to a very different time scale. In such a volatile area of investigation any attempt to synthesise what we know about learning mathematics in the presence of digital technologies runs the risk of having a rather limited shelf-life.

While it might be difficult to predict precisely what even the relatively near future will bring in terms of representational and communicational affordances, it is possible to point to some trends in research and practice in digital technology use. Indeed in the last 10 years there have been a number of projects involving such reviews, with varying emphases including implementation issues (see, for example, Julie et al. 2009; Assude et al, 2009; Wong, 2003), research associated with the use of digital technologies in particular topics (e.g. Guin, Ruthven, Trouche, 2005; Laborde et al., 2006; Ferrera et al., 2006; Thomas and Chinnappan, 2008; Thomas, Monaghan and Pierce, 2004) and considerations of developments in the research agendas associated with digital technology use (Blume and Heid, 2008; Laborde and Straesser, 2010; Lagrange et al. 2003; Healy and Kaput, 2008). The book resulting from the 17th ICMI Study on technology use in mathematics education (Hoyles and Lagrange, 2009), presents a recent state of the art of research into the design of technology-integrated environments and curricula, and into the teaching, learning and assessing of (mainly school) mathematics in the presence of digital technologies.

Amongst the major themes addressed in such reviews, it is possible to identify some points of agreement. One such issue is that, despite the numerous initiatives to introduce digital technologies into the educational systems of many different countries across the world (and the considerable financial investment that has accompanied them), their use continues to be limited in the majority of mathematics classrooms worldwide. A second point of agreement indicates the relative paucity of systematic studies focussing specifically on aspects involving mathematics teachers in educational scenarios involving digital technologies, with much more attention having been directed towards the learner. A third issue relates to ways that digital technologies offer new ways to support problem-solving, experimentation and inquiry. A fourth point emphasises the co-evolution of thinking, tools and knowledge, and the growth of theoretical frameworks which posit reciprocal relationships between—or mutual shaping of—mathematical thinking and technological tools.

The group's challenge and the strategies adopted

It was in this context that this DG was proposed. The central aims of the DG were to: identify possible changes that are brought about by the use of technology within mathematics education; and to consider different factors which contribute to the debates surrounding the integration process and how (or if) technology has impacted on the ways we do, learn and teach mathematics and/or the mathematics itself.

The term "technology" was not used in its most general sense, but as shorthand for "digital technology", with calculator and computer technology given centre stage, along with the new learning spaces and virtual mathematics education communities that they are engendering. This decision was taken in order to promote a focussed discussion, but was not intended to indicate that other kinds of technology are not associated with potential changes to learning ecologies. By focussing on the effects of the current explosion in digital technologies, it is possible, or perhaps even likely, that windows into the mediation on mathematical ideas and practices by different material and semiotic tools, digital or not, will be opened.

In order to promote informed discussion during the group sessions, the following four questions were suggested as starting points prior to the conference.

- *What are the nature and goals of mathematics education and should we rethink the goals in an ever-evolving technological world?*
- *What are the new opportunities that digital technologies offer to mathematics education?*
- *What are the consequences (both positive and negative) that are likely to result from the use of technology in mathematics education and how do these vary in different countries and learning contexts?*
- *Will the use of technology in mathematics education permit a more democratic and universal access to the development of certain sorts of mathematical insight and competencies or will it contribute to an undermining of the mathematical literacy of the student population?*

These questions were posted on the DG Web-site (http://dg.icme11.org/tsg/show/28) and contributions from the mathematics education community were invited. Accepted contributions were also posted and, to stimulate further pre-conference preparation, contributors were also invited to comment specifically on relationships between their own papers and those of others, stressing in particular the four focus questions.

At the congress itself—in line with the ICME characterisation of DGs—it was decided not to structure the discussions around the oral presentation of individual contributions. Instead, the group participants were invited to form subgroups, each of which would address one of the four questions posed. The DG was attended by approximately 65 participants, representing 23 countries. The group split into seven subgroups, with two groups discussing each of questions 1, 2 and 3 and question 4 selected by one group. During the remainder of the first group session, each subgroup worked on the elaboration of a poster to representing their combined ideas on their chosen question. At the beginning of the second session, all the posters were displayed. The second session was split into two parts. In the first, participants added written comments to the posters. In the second, all the participants working on the same question came together to produce a summary presentation based on their subgroup discussions, posters and the comments these had received. These presentations comprised the agenda for the third and final session.

Pre-congress contributions

A total of 16 papers were accepted (see http://dg.icme11.org/tsg/show/28), with four sectors of mathematics education represented: pre-school/primary school education (Highfield and Mulligan, Australia); secondary/high school education (Miyazaki, Kiniho, Arao and Ogihara, Japan; Kemphaus, USA; Baldin, Brazil, Alfonzo and Long, Venezuela & USA); university mathematics (Jarvis, Lavicsa, Buteau, Canada & UK; Nguyen, USA; French, USA); and teacher education (Lins, Brazil; Costa, Brazil; Baldin, Brazil). Some of the contributions concentrated on the discussion issues in the context of specifically designed software environments (Mattos, Guimarães, Barbastefano and Moraes, Brazil; Cuevas Vallejo, Martínez Reyes, Pluvinage, Mexico & France) or tasks (Yang, USA), while in others the main focus was directed towards particular theoretical approaches (Haapasalo, Finland; Bicudo and Rosa, Brazil; Herrera, Preiss and Riera, Chile).

It was noted that much more attention was given in the papers to new ways of doing mathematics with digital technology, than to ways of doing new mathematics and thus the second and the third questions received most attention. In line with the findings of the research reviews cited above, contributions stressed that digital technologies supported tasks that: are open; involve interaction with real world data; are experimental; are collaborative; involve movement between different representations; and permit early encounters with powerful mathematical ideas. The affordances related to auto-assessment were also mentioned along with new possibilities to do mathematics at a distance "holding hands with someone on the other side of the world" as Bicudo and Rosa put it. The changing demands on teachers was also mentioned, and teachers were described variously as mediators, moderators, designers and researchers with more access to more detailed traces of the students' solution processes.

Some results from the discussion sessions

Many of the participants commented on the considerable overlap between the questions and that some issues emerged in almost all the subgroup discussions. There were some points in which different participants held very different views. For example, for the first question posed, one subgroup concluded that "the goals of mathematics education should change to reflect the new mathematics content and methodologies we can do with new technologies" while another subgroup argued that since the goals of mathematics education "largely depend on goals of education in general" then goals probably neither will change, nor should change, because of technology. One of these two subgroups argued that digital technology should be a leading factor in determining "not only what should be taught in the curriculum but also how the mathematics should be taught", and made specific suggestions for change—most notably a plea to "throw out symbolic algebra to make room for data analysis" (a similar suggestion was made by those discussing the fourth question). The other group focused on changes to doing mathematics in the presence of digital technology (including more connections between mathematics inside and outside of formal educational settings, and increased learner autonomy).

In contrast to the subgroups discussing the first question, those who concentrated on the second found close proximity in their ideas. They summarised their discussions by stressing the experimental possibilities brought about by digital technology, illustrated with examples showing the linking of multiple dynamic representations, from contexts both inside and outside mathematics. They also suggested that the set "technology-teaching-learning-mathematics" represents a dynamic totality, using the metaphor of a jigsaw in which pedagogical concerns, issues of access, mathematical knowledge, and familiarity with tools, fit together to compose a coherent whole. The idea of the computer as a potential window into the thoughts of others was stressed—even when those others might not be physically present. The subgroups discussing the third question covered some of the same ground, citing the gains associated with experimentation, with multiple representations, along with the new possibilities for visualisation and concretisation. Alongside, they expressed concerns related to "hot-wiring", that is, the possibility of jumping quickly to false conclusions, and poor retention of critical facts and skills. They also raised questions of time (needed to learn new syntax, for example), pressure to keep up with ever changing tools and inequities in access within, and across, countries. They agreed that the goals of mathematics education need to change, but were worried about who should have the leading voice in making decisions about how, why and when. As such their concerns moved in the direction of the fourth question. The subgroup that chose this last question also saw teacher preparation as a vital, though complex, issue. The participants argued that "the use of technology does not respond by itself for the development of mathematical insights and competencies" and questioned the divide between the digital technology used by many people outside school and which is not yet exploited to change school cultures.

A final reflection on the discussions

The DG brought together an extremely diverse set of participants, from different countries, and with very different experiences of research, teaching and technologies. The level of commitment to the discussion was impressive—although inevitably, the collective result was that more questions were raised than answers reached. This suggests that this is a debate that will continue and we hope that the issues raised during the activities of the discussion will have provided food for thought for all those who participated in them.

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