TSG 19: Research and Development in Problem Solving in Mathematics Education

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1. Introduction

The structure set up to discuss the research agenda and developments in mathematical problem solving was structured around five questions:

1. What features, principles, and ideas are relevant to characterise, rationalise, and relate problem solving approaches to teachers and students' development or constructions of mathematical knowledge?
2. What have been the themes or areas of study in this domain and the extent to which the research results have contributed to frame teachers' and students education and curriculum proposals?
3. To what extent have the developments and availability of digital technology influenced the research and instructional agenda in mathematical problem solving?
4. How and to what extent research results and curriculum proposals based on problem solving, discussed in international forums, have influenced national or local agendas?
5. What are the themes and research directions that are relevant to consider and discuss for future direction in mathematical problem solving?

Thus we report the issues and results that were addressed in both the written reports submitted pre-submitted by participants and the themes addressed in discussions during the sessions. To organise the themes, we have grouped them in four main categories: Problem solving foundation in which we discuss issues related to the characterization of mathematical problem solving and principles associated with research and instruction; research related to cognitive, metacognitve, social, and affective aspects of problem solving approaches; research and development in problem solving approaches that enhance the use of digital technology, and future directions in mathematical problem solving.

2. Research and development in mathematical problem solving

The participants recognised that there might be different ways to engage and promote problem solving environments in order to frame and foster students' mathematical thinking. However a distinguishing feature of the problem solving approach is the inquiring behaviour of learners to reconstruct mathematical knowledge. That is, the students' interaction with a problem or mathematical content is seen as an opportunity for the learners to pose, represent, and pursue questions. We summarise the themes and results that describe the research of this domain.

2.1 Problem solving foundations.

The different interpretations of problem solving in the last four decades have transformed this domain into a complex endeavour involving disciplinary knowledge, cognitive and metacognitive strategies, dispositions, beliefs and affective dimensions, and skills. To deal with this complexity, it is necessary to clarify the role played by problem solving activities in the students' construction of mathematical knowledge. In contrasting previous research in this domain, Lesh, English, and Fennewald (2008) pointed out several shortcomings related to the conceptualisation of problem solving that have limited its development and impact in practice. One is to emphasise that problem solving competencies can be increased by first mastering key concepts, second problem solving strategies, and later learning to use them to solve problems.

Another shortcoming is the emphasis on solving or discussing only school problems (word problems) and paying little attention to problems encountered outside of school. Lesh, et al, also criticised the lack of tools to reliably observe, document, or measure understanding and abilities that contribute to problem solving expertise. They also observed the lack of theory development to frame research that incorporate problem solving experiences that occur beyond the classrooms. As a way to overcome those shortcomings, they propose to orient research around questions such as: (a) What does it mean for students to "understand" relevant heuristics, strategies, beliefs, dispositions, or metacognitive processes? (b) What is the nature of students' early understandings of relevant heuristics, strategies, beliefs, dispositions, or metacognitive processes? (c) How (or in what ways) do students' early understandings develop? Santos-Trigo (2007) shows that not only have the themes and research questions in the problem solving agenda changed, but also the research methods used to investigate those questions. In particular, recent developments in digital technology offer diverse opportunities for learners to represent and explore mathematics problems; and as a consequence, conceptual frames used to explain the students' construction of mathematical knowledge need to be adjusted.

2.2. On the nature of mathematical problem solving

Research results in mathematical problem solving have widely documented the role played by cognitive and metacognitve strategies, conceptions of mathematical knowledge and learning, the affective component, social interactions, and contextual variables in students' learning of mathematics (Lester & Kehle, 2003). An important issue addressed during the \ Congress was to recognise that current development and changes in today's world demands that the nature of problem solving activities includes those situations that deal with complex systems. In this perspective, Lesh & Zawojewski (2007) define problem solving as:

> A task, or goal-directly activity, becomes a problem (or problematic) when the "problem solver" (which may be a collaborating group of specialists) needs to develop a more productive way of thinking about the given situation (p. 782).

In this context, it is important to recognise that students' problem solving proficiency is developed gradually as a result of interacting with a variety of mathematical tasks embedded in diverse contexts. For example, tasks situated in contexts that demand that students to construct a model to decide how to choose a school's soccer team depends on the consideration of relevant information associated with team members' performance. That information is crucial to represent, operate and analyse the model to evaluate its strength and limitations. To approach a problem situated in a mathematical context often requires that the problem solver uses a proper representation to explore mathematical relations. In this process, the problem solver or student formulates conjectures, look for ways to support them, examines counterexamples, and communicates results. Both ways to deal with these types of tasks are key elements for the students to develop problem-solving experiences. Chapman (2008, p166) identifies relevant instructional practices that are crucial to prepare prospective teachers to foster problem-solving activities:

> (1) Exploring others as problem solver: e.g., the prospective teacher works with a child or peer to observe, interview, and document information about this child or peer as a problem solver. (2) Exploring self as problem solver: i.e., inquiring into one's thinking, learning and instructional practices and developing ability to monitor and to control one's activities when solving problems. (3) Exploring nature/structure of problems. (4) Solving challenging problems individually and in small groups without external assistance, e.g., to develop awareness of strategies and skills for solving problems. (5) Posing problems. (6) Comparing self with others, e.g., peers, students, theorists. (7) Formulating instructional model for problem solving. (8) Exploring self as facilitator of problem solving, i.e., to develop an understanding of the teacher's role in facilitating students' problem solving.

In addition, it was recognised that the use of computational tools offers an opportunity for students to represent models or problems information in different ways, including dynamic representations, and as a consequence the students can explore and reason about the problems beyond approaches that rely on the use of paper and pencil reasoning.

2.3. On the use of computational tools.

The developments and the availability of computational tools have influenced not only the development of the discipline but also the way students construct their mathematics knowledge. In this context, some questions that were addressed in the TSG involved: How can we effectively use technologies (for example, internet, calculators, computers) to facilitate problem solving? How can we effectively use technology to advance problem-solving research? What types of mathematical reasoning, including mathematical arguments, do students or problem solvers develop as a result of using various computational tools? What types of strategies and problem representations become important in problem solving environments that promote the use of computational tools? Barrera & Reyes (2008) showed that a dynamic representation of a problem situated in a mathematical context can become a departure point for the problem solver to identify interesting mathematical relations. Those relations or conjectures emerge from exploring behaviours of some objects as a result of moving other mathematical objects within the representation. In addition, the use of the tools offers students the opportunity of examining those relations visually, numerically, and algebraically. In this process, the problem solving approaches enhance mathematical activities such as quantifying parameters (distances, perimeters, areas, etc.), generating loci of mathematical objects, and exploring the behaviours of those objects both empirically and by using algebraic methods.

2.4. Future directions

In general, several curriculum proposals around the world recognise that problem solving activities play an important role in structuring and orienting the students' construction of mathematical knowledge (Törner, Schoenfeld, & Reiss, 2007, NCTM, 2009); however, the multiplicity and a variety of interpretations associated with problem solving that support curriculum projects and research agendas make it necessary to reflect on the fundamentals that characterise any problem solving approach. In this context, the themes that were judged to be relevant and urgent to address by the international problem solving community involve:

i. To what extent do problem solving approaches that promote the students' systematic use of digital tools help them construct mathematical knowledge?

ii. What types of adjustments to conceptual frameworks are needed in order to explain the students' development of new knowledge?

iii. How can conceptual and theoretical developments in problem solving be contrasted with others' theoretical frameworks used in mathematics education?

iv. How can research results in problem solving be used to frame curriculum proposals and instructional approaches?

v. What is the role of problem solving in the education and professional developments of prospective and practicing teachers?

3. The need to activate a problem solving community

An emerging issue that permeated the discussions was that the problem solving community is made of different groups with different aims and interests. This community includes active researchers whose academic agenda involves both theoretical and practicing themes and developments in problem solving. It also includes teachers who show an interest in implementing problem solving approaches in their classrooms and look for ideas to frame their practices around problem solving activities. In particular, teachers' discussions focus on actions to reduce a long list of content and to concentrate on problem solving activities in order to study key concepts deeply. A relevant questions posed here was: What fundamental mathematical ideas and processes should be central in curriculum proposals that promote problem-solving approaches? Another teachers' interest is to address the role of students' international assessments (PISA, TIMMS) in problem solving approaches. That is, to what extent is the mathematics and ways of reasoning involved in those international assessments consistent with problem solving approaches? For example, a problem solving approach values and promotes students thinking of different ways to solve problems and formulating new questions to extend the initial state. And these activities are difficult to evaluate in terms of students' answers to questions given in a limited time. Some countries whose students have shown high scores in international assessment have taken the initiative to engage their students in problem solving activities to deal with non-routine problems. For example, Doorman, Drivers, Dekker, van den Heuvel-Panhuizen, de Lange, and Wijers (2007) report that in the Netherlands there has been a serious attempt to implement a problem solving oriented curriculum structured around realistic mathematics. This involves changes in textbook materials. In Japan, a similar movement is apparent in some textbook problems that encourage students to go beyond reporting a final answer and support their solutions by diverse types of arguments (Takshashi, 2008).

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