

## **TSG 23: The role of the history of mathematics in mathematics education**

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### **The history of mathematics in mathematics education in the 100 years of ICMI**

ICME-11 was held in 2008, the centenary of ICMI, so it was a good occasion to reflect on the themes that have emerged in mathematics education (Furinghetti & Giacardi, 2008). One of those themes is the role of history in mathematics teaching and learning. The relevance of this issue is not surprising, since the first ICMI president, Felix Klein, accorded history an important place in his own work. Schubring (2011) in his Regular Lecture at ICME-11 noted Klein's use of the genetic principle to guide his approach to teaching mathematical topics. One of the 53 reports produced by the German subcommission of ICMI before 1920 was Martin Gebhardt's survey on the use of the history of mathematics in mathematics teaching in Germany. Later, David Eugene Smith supported a historical perspective in the teaching of mathematics and authored books suitable to this purpose. After the Second World War, when ICMI was re-established, *L'Enseignement Mathématique* (1959. s. 2, 5, 284-290) reported on a document by Yasuo Akizuki, which emphasised the importance of the history of sciences, and of mathematics in particular, at all curricular levels.

At the 1972 ICME, a Working Group on "History and pedagogy of mathematics" was organised by Phillip S. Jones and Leo Rogers. In 1976, during ICME-3 at Karlsruhe, ICMI established the International Study Group on the Relations between the History and Pedagogy of Mathematics (HPM) as an affiliate to ICMI, see (Fasanelli & Fauvel, 2006).

Since then, things have evolved: HPM has developed a network of communication and meetings, elaborating the study of the role of the history in mathematics education. Proceedings, books, and special issues of journals have been published on this subject. In particular, the birth of the HPM Study Group fostered the tradition of having special activities on the role of the history in mathematics education in the program of the ICMEs. The TSG 23 at ICME-11 is one of these activities, together with talks and workshops presented during this Congress.

### **The role of history of mathematics in mathematics education**

We identify the following possible aims for the introduction of the history of mathematics into mathematics education

- to humanise mathematics education by conceiving of it as a historical, social and cultural production, and as a set of social activities which are related to other social activities;
- to help students' understand the meanings of aims, values, concepts, methods, and proofs in different social practices involving mathematics;
- to develop learners' feelings of citizenship, contextualising school-based mathematical social practices in a historico-critical point of view and maintaining an open attitude towards the study of mathematical practices in different geopolitical, institutional and chronological contexts.

Even if mathematics teachers agree that these aims are fundamental in mathematics teaching, there is evidence of certain reluctance about the introduction of the history of mathematics in the classroom, in spite of decades of encouragement by curriculum developers and researchers. Siu (2006) has summarised the difficulties of this introduction in 16 points centred on poor teacher preparation, the difficulties of access to, and use of, original sources, and the pressures of the school systems. The TIMSS 1999 Video Study seems to show that

history of mathematics does not constitute an important part of teaching in the seven visited countries (see Smestad, 2006).

The following points were proposed for discussion of the opportunities and problems linked to the use of historical elements in the teaching of mathematics.

- What kind of proposals, strategies and pedagogical practices have been useful to the establishment of fruitful dialogue between history and mathematics education in different countries, contexts and levels, chiefly in mathematics teacher education?
- How and why the integration of history of mathematics in mathematics education has been differently valued in the school curricula of different countries and regions?
- What are the political, philosophical, sociological, anthropological, psychological and linguistic perspectives for the integration of history in mathematics education?
- To what extent could the production of new and more enlightening histories of mathematics and mathematics education contribute and promote new ways of establishing dialogues between history and mathematics education?
- What is the place of the history of mathematics in textbooks of different countries?
- What place do the historical mathematics textbooks occupy in the mathematics' teachers' education in different countries?
- How can the ICT help to promote dialogues between history and mathematics education?
- Which media can promote dialogue between history and mathematics education?

### **The programme of TSG 23**

The programme of TSG 23 consisted of the presentation of invited and contributed papers, and open discussion. We identify the following main themes:

- Activities with history aimed at raising students' motivation (Charbonneau; Smestad)
- Teachers' development programmes based on the history of mathematics (Lawrence).
- The use of history to explore and overcome students' difficulties in connection with specific topics (Kourkoulos & Tzanakis; Milevicich & Lois).
- Reflections on methodological issues for research in the field (Jankvist).

#### ***Invited papers***

L. Charbonneau: *Astronomical and mathematical instruments as pedagogical tools.*

In the province of Québec, history of mathematics has been incorporated into the curriculum. In the "repères culturels" sections of the curriculum, references to the use of instruments are often made. This paper illustrates some uses of such instruments in the classroom, focussing on the construction of some 16th, 17th and 18th century instruments and the importance of astronomical or geometrical models behind such construction.

U. T. Jankvist: *On empirical research in the field of using history in mathematics education.*

This paper focuses on the role empirical research may serve in the discussion of why and how to use history in mathematics education. This is exemplified by referring to two studies in the Danish upper secondary mathematics programme. The paper illustrates how both the research design and research methodology of these two studies were dependent on using history as a goal rather than history as a tool.

M. Kourkoulos & C. Tzanakis: *Contributions from the study of the history of statistics to understand students' difficulties to grasp the concept of variance.*

This study focuses on difficulties often encountered by students trying to understand the concept of variance. A historical approach helps to bring out the deep character of the pedagogical issues involved. Specifically, a historical approach brings out the following

- (a) The importance of the contexts in which these concepts arose, that is, in the study of measurement error in astronomy and geodesy in the 18<sup>th</sup> and early 19<sup>th</sup> centuries.
- (b) The relation between statistics and physics included the kinetic theory of gases and

statistical mechanics, leading to an understanding of concepts such as the absolute temperature of ideal gases and Brownian motion.

In introductory statistics courses, it is common to use examples related to social phenomena. The analysis of students' behavior shows that (i) it is difficult for students to get a coherent meaning of variance only on the basis of situations related to social phenomena; (ii) such a restriction can activate important epistemological obstacles. Further didactical analysis suggests that one of the most promising ways may be based on simple physical models.

B. Smestad: *Student projects on history of mathematics.*

From 2003-06, students were assigned a project creating historical materials for the class. This paper looks at the way they chose approach this, to understand their conceptions of history of mathematics, and its role in mathematics education. Some students had a limited concept of "history of mathematics" and of what this could bring to their mathematics teaching. This should be taken into account when trying to promote historical perspectives in the classroom.

### ***Contributed papers***

L. Haapasalo: *On instrumental genesis within procedural and conceptual thinking.*

Technology has caused a holistic change in the way we think, plan, work and evaluate in a modern society. Thus educators are forced to shift their views of the learning of mathematics. On the other hand, fostering students' problem-solving abilities means emphasising the genesis of heuristic processes and students' possibilities to develop mathematical intuition. This can be realised within constructivist views of teaching and learning: concepts and procedures can be constructed by students themselves, and the other way around. Solving authentic problems using technology promotes both of these goals and allows us to gain from students' natural activities outside the classroom. This paper describes a ClassPad project from the viewpoint of problem-solving, emphasising the making of informal rather than formal mathematics on the basis of a 'Minimalist Instruction' approach and within the framework of eight activities.

S. Lawrence: *History of Mathematics making its way through the teacher networks – Professional learning environment and the history of mathematics in mathematics curriculum.*

This paper describes a 2006-08 project aimed at introducing the history of mathematics into the curriculum through a collaborative teaching practice involving both primary and secondary schools in the South East of England. It looks at the uses of the history of mathematics to initiate the increased motivation of students, and the creation of a conceptual landscape for the teachers' continual professional development.

L. Milevicich & A. Lois: *The historical and cognitive development of calculus ideas.*

It is not easy to identify ways to implement history of mathematics in education, as it depends on: the educational level; the subjects and their specific problems; the historical knowledge of the teacher including interest in interdisciplinary work, predisposition and ability to perform didactic transposition, adaptation, reconstruction, recreation and transformation of historical knowledge. This paper presents a theoretical discussion of parallelism between the historical development and the cognitive development of mathematical ideas.

### ***Written contributions***

The participants of TSG 23 had also the opportunity to discuss the following written contributions by colleagues who were not able to attend ICME-11 conference:

- A. Cesar de Mattos: *The process of recognition in the history of mathematics*
- D. Pareja-Heredia: *The huge gap between math education and the front of mathematics*
- S. Mrabet: *The development of Thales theorem throughout history*
- G. Wielewski: *Two cultures of mathematics in historical and educational perspective*

### **Final remarks**

In the four sessions, we addressed the discussion questions through the participation of: mathematics education researchers, historians of mathematics, and teachers. The participants

brought to the discussion the influence of their different cultures, school systems, and school practices. The role of the history of mathematics inserted in mathematics teaching acted as a catalyst to the general problems of mathematics teaching.

We state a list of themes that need to be considered in the future research.

- Theoretical and conceptual frameworks for including history in mathematics education.
- The role of history of mathematics at primary and secondary level, from the cognitive and affective points of view.
- The role of history of mathematics in pre- and in-service teacher education, from the cognitive, pedagogical, and affective points of view.
- Parallelism between the historical and cognitive development of mathematical ideas.
- Classroom research on integrating original sources, and their effects.
- Surveys on uses of history in curriculum, textbooks, at all levels.
- Design and assessment of teaching/learning materials on the history of mathematics.

The participants agreed on the need for empirical research. We encourage researchers and teachers to undertake teaching experiments and consequently modify the initial design. We believe this will result in gains in teachers' satisfaction and the spread of mathematical culture.

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