

## Third Stage of Schooling (Grade 7~9)

### I. Numbers and algebra

#### (I) Numbers and expressions

##### 1. Rational numbers

- (1) Understand the meaning of rational number. Able to use the points on the number line to represent rational numbers. Able to compare the magnitude of rational numbers.
- (2) Make use of the number line to understand the meanings of opposite numbers and absolute values, master the methods to find opposite numbers and absolute values of rational numbers, know the meaning of  $|a|$  ( $a$  is a rational number).
- (3) Understand the meaning of power, master addition, subtraction, multiplication, division, power and simple mixed computation of rational numbers (mainly within three steps).
- (4) Understand operation rules of rational numbers, able to use operation rules to simplify operations.
- (5) Able to deploy operations of rational numbers to solve simple problems.

##### 2. Real number

- (1) Understand the concepts of square root, radical of a number (i.e. positive square root of a number), and cube root, able to use the radical sign to represent the square root, radical, and cube root of a number.
- (2) Familiarize with that power and root are both inverse operations, able to use the square root operation to evaluate the square root of a number within 100, Able to use the cube root operation to evaluate the cube root of a number (corresponding to negative integers) within 100. Able to use a calculator to evaluate square root and cube root.
- (3) Familiarize with the concept of irrational number and real number. Know the one-one correspondence relationship between real numbers and points on the number line, able to evaluate the opposite number and absolute value of a real number.
- (4) Able to use a rational number to estimate the approximate region of an irrational number (Refer to example 47).
- (5) Familiarize with approximate numbers and able to use a calculator to approximate an answer so as to solve practical problems, and give an approximate value to the result obtained in accordance with the requirements of the question.
- (6) Familiarize with the concepts of radical expression of second degree, and the simplest expression of second degree, and the addition, subtraction, multiplication and division rules, able to use these concepts and operation rules to carry out simple arithmetic operations (Refer to example 48).

##### 3. Algebraic expressions

- (1) Within realistic contexts, familiarize with the meaning of algebraic expressions, move a step further to familiarize with the meaning the meaning of using a letter to represent a number (Refer to example 49).

- (2) Able to analyze simple quantitative relations of concrete problems, and to use algebraic expressions for their representation.
- (3) Able to evaluate algebraic expressions, able to search information in accordance with specified problems, Able to find the necessary formula, and to substitute concrete values for its evaluation.

#### **4. Integral and Fractional Expression**

- (1) Familiarize with the meaning and basic properties of integer index power, Able to use scientific notation to represent a number (including display with calculator).
- (2) Familiarize with the concept of integer expression. master the rules of merging similar terms and removing parentheses, able to carry out addition and subtraction operations for simple integer expressions, able to do multiplication operations for integer expressions (the multiplication of polynomials is merely multiplication of linear or multiplication of quadratic by linear).
- (3) Able to derive the multiplication formula:  $(a+b)(a-b) = a^2 - b^2$ ,  $(a \pm b)^2 = a^2 \pm 2ab + b^2$ , Familiarize with the geometric background of the formulas, and Able to do simple computations using the formulas (Refer to example 50).
- (4) Able to use common factor extraction method and of formula method (direct use of formulae can not be more than twice) to carry out factorization (the indexes is a positive integer).
- (5) Familiarize with the concept of fraction expression and simplest fraction expression, able to use the basic properties of fraction expression to do reduction, as well as reduction of fraction expression to a common denominator, Able to do simple addition, subtraction, multiplication and division operations of fraction expression.

### **(II) Equations and inequalities**

#### **1. Equation and system of equations**

- (1) Able to construct an equation in accordance with the quantitative relations of concrete problems, experience that equation as a mathematics model is effective to depict the realistic world (Refer to example 51).
- (2) Experience the process of solving equations (Refer to example 52).
- (3) Master the basic properties of equalities.
- (4) Able to solve equation of first degree in one unknown,, as well as to simplify these into fraction equations of first degree in one unknown.
- (5) Master the methods of “substitute to eliminate unknowns” and “add and subtract to eliminate unknowns”, able to solve system of equation of first degree in two unknowns .
- (6) \*<sup>1</sup>Able to solve simple system of equation in three unknowns.
- (7) Familiarize with the methods of changing an incomplete square expression into a complete one, and able to use the method to solve quadratic equations in one unknown with number coefficients, and able to use formula method and method of factorization as well.
- (8) Able to examine whether an equation has real roots and if two real roots are equal by using

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<sup>1</sup> *When a content is labeled with \*, it will not be the content that included in the final examination*

the discriminate for roots of quadratic equation in one unknown.

- (9) \*Familiarize with the relations between the roots and coefficients of quadratic equation in one unknown.
- (10) Able to examine whether the result obtained is reasonable or not in accordance with the practical meaning of the concrete problem.

## **2. Inequality and System of Inequalities**

- (1) Familiarize with the meaning of in inequality in accordance with concrete problem (Refer to example 57), and to explore the basic properties of inequality.
- (2) Able to solve inequality of first degree in one unknown with number coefficients and able to indicate the solution set on the number line. Able to solve system of inequalities involving two inequalities of first degree in one unknown, and to use a number line to ascertain the solution set
- (3) Able to construct an inequality of first degree in one unknown in accordance with quantitative relations of concrete problems, and to solve simple problems.

## **(III) Functions**

### **1. Function**

- (1) Explore the quantitative relations and variation patterns in practical situation, familiarize with the meanings of constant and variable.
- (2) In accordance with practical example, familiarize with the concept of function and three methods of representation. Able to give concrete examples of a function.
- (3) Able to make use of graphs to carry out analysis on the function relations of simple practical problem (Refer to example 54).
- (4) Able to determine the domain of independent variable of a function for simple practical problem, and able to evaluate the value of the function.
- (5) Able to use appropriate function representation methods to depict the relationships of variables of some simple practical problems.(Refer to example 55).
- (6) Able to make use of analysis of function relations to discuss preliminarily the variations of variables.(Refer to example 56).

### **2. First degree function**

- (1) Coupled with concrete situations, experience the meaning of first degree function, able to ascertain function expression of a first degree function in accordance with known conditions (Refer to example 57).
- (2) Able to ascertain function expression of a first degree function using the method of “coefficients to be determined”.
- (3) Able to draw the graph of a first degree function, Explore and understand the property of  $y=kx+b$  ( $k \neq 0$ ) based on the explication of this expression and graph of first degree function when  $k > 0$  or  $k < 0$ .
- (4) Understand the direct proportion function.
- (5) Experience the relations between first degree function and first degree equation In two unknowns

- (6) Able to use first degree function to solve simple and practical problems.

### **3. Inverse proportion function**

- (1) Coupled with concrete situation, experience the meaning of inverse proportion function, able to ascertain the expression of an inverse proportion function in accordance with given conditions.
- (2) Able to draw the graph of an inverse proportion function, explore and understand the property of  $y=k/x$  ( $k \neq 0$ ) based on explication of this expression and graph of first degree function when  $k > 0$  or  $k < 0$ .
- (3) Able to use inverse proportion function to solve simple practical problems.

### **4. Quadratic function**

- (1) Experience the meaning of quadratic function through analyzing contexts of practical problems.
- (2) Able to use the method of plotting points on a plane to draw the graph of a quadratic function, able to know the properties of quadratic function by means of its graph.
- (3) Able to use method of complete the square to represent a quadratic function with number coefficients with the expression of  $y=a(x-h)^2+k$ , and able to hereby obtain the vertex coordinates of the quadratic function. And able to tell the opening direction of the graph, as well as to draw the symmetric axis of the graph. Able to solve simple practical problems.
- (4) Able to use the graph of quadratic function to find an approximate solution of quadratic equation in one unknown.
- (5) \* Know that the given coordinates of three non-collinear points can determine a quadratic function.

## **II. Figures and Geometry**

- (1) Properties of Figures

### **1. Point, line, plane, angle**

- (1) Through a variety of real objects and concrete models, familiarize with geometric solids such as plane, line and point.(Refer to example 58).
- (2) Able to compare the lengths of segments, understand the meaning of sum, difference, and middle point of segments.
- (3) Master the basic fact that two points ascertain one straight line.
- (4) Master the basic fact that the segment between two points is the shortest.
- (5) Understand the meaning of the distance between two points and can measure this distance.
- (6) Understand the meaning of angle and able to compare the sizes of angles.
- (7) Familiarize with degree, minute and second, and able to do simple conversion of them. Able to compute addition and subtraction of the angles.

### **2. Intersecting lines and parallel lines**

- (1) Understand the concepts of vertically opposite angle, complementary angle and supplementary angle, explore and master the properties that vertically opposite angles are equal, the

complementary angles of equal angles are equal, as well as the supplementary angles of equal angles are equal.

(2) Understand the concepts of perpendicular and its line segment, Able to use set squares or a protractor to construct a perpendicular from a point to a straight line.

(3) Understand the meaning of distance of a point to a straight line and able to measure this distance as well.

(4) Master the basic fact that through a given point there is one and only one straight line perpendicular to the given straight line.

(5) Familiarize with opposite angles, alternate angles and interior angles on the same edge.

(6) Understand the concept of parallel lines. Master the basic fact that if two straight lines are intersected by a third line, and if the opposite angles are equal, then the two lines are parallel.

(7) Master the basic fact that through a given point outside a straight line there exists one and only one straight line parallel to the given straight line.

(8) Master the properties theorem of parallel lines that if two parallel lines are intersected by a third straight line, the corresponding angles are equal. \*Familiarize with the proof of the theorem. (Refer to example 59).

(9) Able to use set squares and a straight edge to construct a straight line passing through a given point and parallel to a given straight line.

(10) Explore and prove the discrimination theorem of parallel lines that If two straight lines are intersected by a third straight line and if the alternate interior angles are equal (or, the interior angles on the same edge are supplementary to each other), then those two straight lines are parallel. Explore and prove the properties theorem of parallel lines that when two parallel straight lines are intersected by a third straight line, the alternate interior angles are equal (and, the interior angles on the same edge are supplementary to each other).

(11) Familiarize with that if each of the two straight lines is parallel to a third straight line, then those two straight lines are parallel. Lines

### **3. Triangle**

(1) Familiarize with the related concepts of triangle such as interior angle, exterior angle, median, altitude, angle bisector, Familiarize with the stable characteristics of a triangle.

(2) Explore and prove the theorem on angle sum of interior angles of a triangle. Master its corollary. An exterior angle of a triangle is equal to the angle sum of two non-adjacent interior angles. Prove that the sum of any two sides is greater than the third side.

(3) Understand the concept of congruent triangles. Able to recognize corresponding sides and corresponding angles under which two triangles are congruent.

(4) Master the basic fact that if two sides and their included angle are respectively equal, then the two triangles are congruent (Refer to example 60).

(5) Master the basic fact that if two angles and their included side are respectively equal, then the two triangles are congruent (Refer to example 60).

(6) Master the basic fact that if three sides of two triangles are respectively equal, the two triangles are congruent.

(7) Prove the theorem that if two angles of two triangles are respectively equal, and also the opposite sides of both equal angles are equal, then the two triangles are congruent.

(8) Explore and prove the properties theorem of angle bisector that the distances from any point

on angle bisector to two sides of the angle are equal, and conversely that if the distances from a point within an angle to two sides of the angle are equal, then the point is on the angle bisector.

- (9) Understand the concept of perpendicular bisector. Explore and prove the properties theorem of perpendicular bisector that the distances from any point on a perpendicular bisector to two ends of the segment are equal. And conversely that if the distances from the point to the two ends of the segment are equal, then any point is on the perpendicular bisector,
- (10) Understand the concept of isosceles triangle. Explore and prove the properties theorem of isosceles triangle in which two base angles of an isosceles triangle are equal, the height, median on the base, as well as the bisector of the vertex angle coincide. Explore and master the discrimination theorem of isosceles triangle in which a triangle which having two equal angles is isosceles. Explore the properties theorem of equilateral triangle in which each angle of an equilateral triangle is equal to  $60^\circ$ . And also the discriminator theorem of equilateral triangle that a triangle which having three equal angles is equilateral (Or an isosceles triangle that having an angle of  $60^\circ$  is equilateral.).
- (11) Understand the concept of right-angled triangle. Explore and master the properties theorem of right-angled triangle in which two acute angles of a right-angled triangle are complementary to each other, and the median on the hypotenuse of a right-angled triangle is equal to half of the hypotenuse. Master that when two angles of a triangle are complementary to each other, the triangle is a right-angled triangle.
- (12) Explore Pythagorean Theorem and its inverse theorem, and Able to use them to solve some simple and practical problems.
- (13) Explore and master the theorem of “hypotenuse, right angle sides” that is the discrimination theorem of right-angled triangles congruent  
Familiarize with the concept of center of gravity of a triangle.

#### **4. Quadrilateral**

- (1) Familiarize with the definition of polygon, and the concept of vertex, edge, interior angle, exterior angle, and diagonal. Explore and master the formulae of sum of interior angles and sum of exterior angles of a polygon.
- (2) Understand the concepts of parallelogram, rectangle, diamond, and square, and relations among them, Familiarize with the instability of quadrilateral.
- (3) Experience and prove the properties theorem of quadrilateral. A quadrilateral, of which a pair of opposite sides is parallel and equal, or two pairs of opposite sides are respectively equal, or diagonals bisect each other, is a parallelogram. Explore and prove discrimination theorems of quadrilaterals. A quadrilateral, of which a pair of equal and parallel opposite edges, is a parallelogram. A quadrilateral, of which two sets of opposite edges are equal respectively, is a parallelogram. A quadrilateral, of which its diagonals bisect each other, is a parallelogram.
- (4) Familiarize with the meaning of distance between two parallel lines; Able to measure the distance between two parallel lines.
- (5) Explore and prove the properties theorem of rectangle, diamond and square. For rectangle, of which all its four angles are right angles, and the two diagonals are equal in length. A diamond, of which the four sides are equal, and the two diagonals are perpendicular. And their discrimination theorems, a quadrilateral with three angles right-angled, or a parallelogram

with diagonals equal to each other, is a rectangle. A quadrilateral with four sides equal, or a parallelogram with diagonals perpendicular to each other, is a diamond, The Square has all properties of rectangle and diamond.(Refer to example 61).

- (6) Explore and prove the theorem of medians of a triangle.

## 5. Circle

- (1) Understand the concepts of circle, arc, string, central angle, and angle at circumference, Familiarize with the concepts of equal circles and equal arcs. Explore and familiarize with the positional relation between point and circle.
- (2) \* Explore and prove the theorem of perpendicular diameter. Of which, the diameter, which is perpendicular to a string, bisects the string and the two arcs corresponding to the string.
- (3) Explore the relations between angle at circumference and central angle and their corresponding arcs, Know and prove the theorem of angle at circumference and its deduction. Of which, the number of degrees of an angle at circumference is equal to half of the number of degrees of the central angle corresponding to its arc. The angle at circumference, corresponding to a diameter, is a right angle. The string, of which corresponding to the angle at circumference of  $90^\circ$ , is a diameter. The opposite angles of a circle's inscribed quadrilateral are supplementary to each other.
- (4) Familiarize with internal center and ex-center of a triangle.
- (5) Familiarize with the positional relations between straight line and circle, master the concept of tangent, explore the relations between a tangent line and the radius that cuts at the tangent point, and able to draw a tangent line at a point on a circle with triangle square.
- (6) \* Explore and prove the theorem of the length of tangent. Of which, the two tangents to a circle, drawn to pass through a point outside the circle, are equal in length (Refer to example 62).
- (7) Able to calculate arc length and area of a sector.
- (8) Familiarize with the concept of regular polygon and the relations between regular polygon and circle.

## 6 Construction figures using compasses and straight edges

- (1) Able to use straight edges and compasses to complete the following figure constructions :construct a line segment equal in length to a given line segment, an angle equal in size to a given angle, the bisector of an angle, the perpendicular bisector of a line segment, the perpendicular to a given straight line, the perpendicular passing through a point.
- (2) Able to use basic figure construction methods to construct a triangle: construct a triangle given the length of its three sides, two edges and their included angles, or given the size of two angles and their included edge. Construct an isosceles triangle given the base side and altitude standing on the base of a triangle, a right-angled triangle given one right angle edge and hypotenuse.
- (3) Able to draw circles that pass through three points not lying on the same straight line, to draw the circumscribed circle, and internally tangent circle for a triangle, to draw the inscribed square and right hexagon of a circle.
- (4) Familiarize with the reasons of construction of figures using compasses and straight edges without markings, and it is not required to write down the methods of drawing.

## **7, Definition, proposition, theorem**

- (1) Coupled with concrete practical examples, Familiarize with the meanings of definition, proposition, theorem, and corollary.
- (2) Coupled with concrete practical examples, able to distinguish conditions and conclusions of proposition, and Familiarize with the concepts of original proposition and its inverse. Able to recognize two propositions which are inverse to each other. Know that if an original proposition is true, its inverse proposition is not necessarily true.
- (3) Know the meaning of proof and the necessity of the proof (Refer to example 74), know that a proof must be logically (Refer to example 63); know that the proving process can have different forms of expression, know the logical written steps of a proof.
- (4) Familiarize with the function of counter-example and know that the right or wrong of a proposition could be examined by the counter-example.
- (5) Experience the meaning of counter-proving method in accordance with practical examples.

## **(II) Figures and their transformation**

### **1. Line symmetry of figures**

- (1) Through concrete real examples know axis symmetry, and explore its basic properties. For two axis symmetric figures, Understand the property that line segment joining corresponding points are perpendicularly bisected by the symmetric axis (Refer to example 64).
- (2) Able to draw axis symmetric figure of a simple plane figure (point, segment, straight line, triangle,) with reference to given symmetric axis.
- (3) Understand the concept of axis symmetric figures. Explore the axis symmetry property of isosceles triangle, rectangle, diamond, regular polygon, and circle.
- (4) Recognize and appreciate axis symmetric figures in nature and realistic everyday living.

### **2. Rotation of figures**

- (1) Familiarize with rotation of a plane figure and explore its basic properties through concrete practical examples. Explore its basic properties that the distances from a given point to the center rotation is the same as that of the corresponding point after rotation, as well as angles formed by segments connecting points to centre of rotation are equal to that of the corresponding points after rotation (Refer to example 64).
- (2) Familiarize with the concepts of Centro-symmetry and Centro-symmetric figures. Explore its basic properties that for two Centro-symmetric figures, the connecting line of corresponding points passes through center of symmetry, and is bisected by center of symmetry.
- (3) Explore properties of Centro-symmetry of segment, parallelogram, regular polygon, and circle.
- (4) Recognize and appreciate Centro-symmetric figures in nature and realistic everyday living.

### **3. Translation of figures**

- (1) Coupled with concrete practical example, recognize translation and explore its basic properties that segments formed by connecting points to corresponding points are parallel (or on the same straight line) and equal (Refer to example 64).



- (2) Recognize and appreciate the applications of translation in nature and realistic everyday living.
- (3) Use translation axis symmetry and rotation to carry out pattern design of figures.

#### **4. Similarity of Figures**

Familiarize with of proportion, ratio of line segments and proportional segments as well as their basic properties. Familiarize with golden section through practical examples of architecture and arts.

- (1) Recognize similarity of figures through concrete practical examples. Familiarize with similar polygons and similarity ratio.
- (2) Master the basic fact that when two straight lines are intersected by a set of straight lines, the corresponding segments obtained are proportional.
- (3) Familiarize with the discrimination theorem of similar triangles in which two triangles are similar if there are two angles equal for each of them, or two sides proportional and their included angles equal, or its three sides are proportional.. \* Familiarize with the proof of the discrimination theorem as well.
- (4) Familiarize with properties theorems of similar triangles in which the ratios of corresponding segments are equal to similarity ratio, or the area ratio is equal to the square of the similarity ratio.
- (5) Familiarize with homothetic of figures, Able to use homothetic to magnify or reduce the size of a figure.
- (6) Able to use similarity of figures to solve some simple practical problems. (Refer to example 74).
- (7) Through similarity of right-angled triangles to explore and recognize the trigonometric functions of acute angles( $\sin A$ ,  $\cos A$ ,  $\tan A$ ), know the trigonometric functions values of  $30^\circ$ ,  $45^\circ$ , and  $60^\circ$  angle.
- (8) Able to use calculator to compute the trigonometric function values of the given acute angles, and vice versa.
- (10) Able to use acute angle trigonometric functions to solve right-angled triangle, able to use related knowledge to solve some simple and practical problems.

#### **5. Projection of figures**

- (1) Familiarize with the concepts of central projection and parallel projection through abundant practical examples.
- (2) Able to draw the three-view drawing, front, left and top view, of right prism, cylinder, cone and sphere, able to judge the three views of simple objects, and able to describe basic geometric objects in accordance with the three views of the objects.
- (3) Familiarize with lateral face of the net of right prism and cone, able to judge and make three-dimensional models in accordance with the net of the objects.
- (4) Familiarize with the applications of above views and nets in realistic life through practical examples.

### **(III) Figures and their coordinates**

#### **1. Coordinates and positions of figures.**

- (1) Experience and learn further those positions of an object able to be represented by an ordered pair in accordance with practical examples.
- (2) Understand the related concepts of rectangular coordinate system on a plane, able to write down rectangular coordinate system. In given rectangular coordinate system, able to plot the positions of points, and based on the positions of points to write down their coordinates.
- (3) Coupled with practical problems, able to develop appropriate rectangular coordinate system, and describe the positions of objects (Refer to example 65).
- (4) Given a square, able to develop appropriate rectangular coordinate system and write down the coordinates of its vertexes.. Experience and learn that a simple figure can be written down by its coordinates.
- (5) Able to use azimuth and distance to describe relative positions of two objects on a plane (Refer to example 66).

#### **2. Coordinates and movements of figures**

- (1) In a rectangular coordinate system, taking the coordinate axes as symmetric axes, able to write down the coordinates of vertexes of symmetric figure of an original polygon, the coordinates of vertexes of the original polygon being given, and know the relation between corresponding vertex coordinates.
- (2) In a rectangular coordinate system, able to write down the vertex coordinates of the figure of a translated polygon, of which is obtained from a polygon with given vertex coordinates and the polygon is translated in the directions of coordinate axes, and know the relations between the coordinates of corresponding vertexes.
- (3) In a rectangular coordinate system, explore and familiarize with that if a polygon is translated along the two axis directions one after the other to obtain another figure, the obtained figure will have translation relations with the original one. Experience the change of vertex coordinates of the figures.
- (4) In a rectangular coordinate system, explore and familiarize with that when the vertex coordinates of a polygon (one vertex is on the origin point of the coordinates and one edge is on the horizontal axis) are enlarged or reduced for the same times respectively, the corresponding figure obtained is position-similar with the original one.

### **III. Statistics and probability**

#### **(I) Sampling and data analyzing**

1. Engage in activities of data collection, organization, description, and analysis. Understand the process of data processing, and able to use calculator to process relatively complicate data.
2. Experience and learn the necessity of sampling, familiarize with simple random sampling through practical examples (Refer to example 67).
3. Able to make pie chart statistics graphs. Able to describe data intuitively and effectively with statistics graphs.
4. Understand the meaning of mean, able to compute the median, mode, weighted mean, and

familiarize with that they are the representation of the central tendency of data (Refer to example 68).

5. Experience and learn the meaning of representation of data discretion, able to compute the variance of simple data.
6. Through practical examples, understand the meaning of frequency and frequency distribution, able to draw frequency histogram and able to use it to explain the information contained in data.(Refer to example 70).
7. Experience and learn the relations between samples and population, able to use mean and variance of the sample to estimate the mean and variance of the population.
8. Able to explain the statistics results. Able to make simple judgment and prediction in accordance with statistics results, and engage in exchange of ideas.(Refer to example 70).
9. Through table, broken-line chart, tendency chart to, feel the changing tendency of the random phenomena (Refer to example 71).

## **(II) Probability of event**

1. Able to use tabulation or draw tree diagram to deploy all possible results of simple random event, and able to list all possible results of occurrence of specified event. Familiarize with the probability of an event (Refer to examples 72, 73).
2. Know that probability able to be estimated with frequency by means of large quantity of repeated tests.

## **IV. Synthesis and practice**

1. Coupled with practical situation, experience the process of designing and implementing a plan to solve concrete problem. Experience and learn the process of mathematics modeling and solving problems, and try to discover and raise questions during the process.
2. Able to reflect the entire process of participating in activities, able to make a report or small thesis about results and process of research, able to exchange ideas with others, and able to acquire experiences for the further mathematical activities.
3. Through exploration, familiarize with the internal connections amongst the various branches of mathematical knowledge (including other discipline knowledge), understand the related knowledge further, and develop consciousness and capabilities for applications. (Refer to examples 74, 75, 76, 77, 78, and 79).