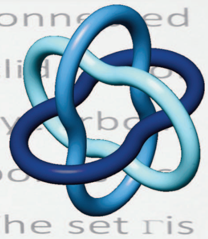
ICM 2010
HYDERABAD • INDIA

REFLEXIONS

The daily newsletter of ICM 2010

August 24, 2010



"I find the subject fascinating..."

VISWANATHAN ANAND: *Nowadays there's access to a lot of tools which can make mathematics much more interesting*

You will be playing against 40 mathematicians simultaneously during the forthcoming ICM. Do mathematicians make better chess players... have you had reason to suspect such a link?

I think they would definitely have the aptitude for it and I think I can have some interesting games. Both areas require a lot of work, a lot of practise… you need to keep on learning. I think in mathematics like in chess, there are many different areas. The broader the chess player you are, the easier it is to be competitive and the same seems to be true of mathematics–if you can find links between different branches of mathematics it can help you resolve problems. In both mathematics and chess you study existing theory and use that to go forward.

You enjoy reading about mathematics. When did you develop this liking? Did you like mathematics at school?

I was reasonably interested in mathematics in school. Typically what happens is… when you start playing chess it takes up a lot of your attention. But about 10 years ago I found that the internet is very good to start learning about a lot of subjects. I am interested in astronomy also. The internet gives you access to a lot of material and it's fun to sit and read. I go to something like Wikipedia and look at different topics… I find the subject fascinating. I like to read about concepts and mathematicians. There are also books. Of course I am an amateur -I enjoy reading about it but don't have the background to work on it on myown…

What do you think about the way mathematics is taught at school? What were your teachers like? Is there anything that you think could be done differently in the way mathematics is taught at school?

Well, nowadays it seems there's access to a lot of tools which can make math much more interesting. I wouldn't say school is bad — teachers have a few months to get the subject across and then you have to pass the exams.

I don't think they could have done it differently. But I think on the whole, the subject could be presented differently. Because it is very gripping once you start reading about it. Nowadays, as a consumer if you like, you find a lot of people have taken a lot of effort to make it fun and enjoyable. And then it's a blast. But in school they don't have the context to do that. Perhaps they should redesign it. But this has to be done at a fundamental level. I don't think it's up to individual teachers, of course they can make some difference.

Depending on how it's presented you could fall in love with it or it can turn you away… So much of modern life is built on mathematics and I'm not sure school really prepares you to enjoy this fully. Things like the financial markets — a proper grounding in mathematics could help the common man. I believe that if people are more familiar with mathematical concepts… it can help deal with modern life which is increasingly complex.

Are there any specific branches of mathematics that you find interesting? Why?

There used to be a column in the New York Times on mathematics by Steven Strogatz. I found that very interesting. Basically it's not a specific area of mathematics… if anything is explained well, it is very enjoyable. It used to cover a lot of topics. The column seems to be taking a break…

Do you think you would have liked to be a mathematician if not a chess player?

Yes … it is definitely something I wish I could have done more of.

Rosalind Ezhil K.
(Rosalind Ezhil used to be a Ramaseshan Fellow at the Indian Academy of Sciences).

This interview appeared in *The Hindu*, August 2, 2010

The names of people who will play Anand:

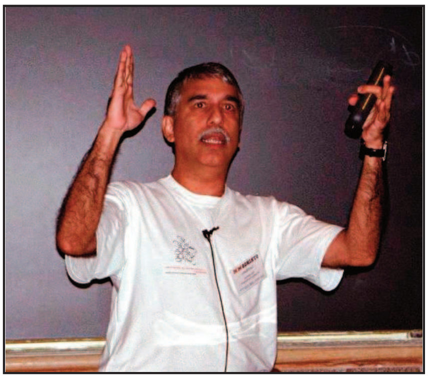
Zoran Rakić	SERBIA
Jitender Singh	INDIA
Jayaraman Boobalan	INDIA
Abhishek Parab	INDIA
Deepjyoti Goswami	INDIA
Prajapati Udayan Maneklal	INDIA
Pravas Karuvachery	INDIA
Palden Ball	U K
John Bellingham	AUSTRALIA
Gergely Röst	HUNGARY
Muralidhara V N	INDIA
Killampalli Srinivasa Rao	INDIA
Farhad Merchant	INDIA
Carles Casacuberta	SPAIN
Enkhbat Rentsen	MONGOLIA
Meetal Mahesh Shah	INDIA
Raghavendar K	INDIA
Srikar Varadaraj	INDIA
Gaven John Martin	NEW ZEALAND
Paul Balmer	UNITED STATES
Alexey Borodin	UNITED STATES
Chandrasheel R Bhagwat	INDIA
Anant R Shastri	INDIA
Gagandeep Singh	INDIA
Indu Bala Bapna	INDIA
Paul Alton Hagelstein	UNITED STATES
Kannan Soundararajan	UNITED STATES
Uwe Kähler	PORTUGAL
Christian Baer	GERMANY
Peter Csikvári	HUNGARY
Endre Csóka	HUNGARY
Francisco J F Polo	SPAIN
Ilona Gyöngyi Bankuti	HUNGARY
Daniele Garrisi	KOREA, REP.
Chris Connell	UNITED STATES
Srikant v (Infosys)	INDIA
KrishnaChaitanya (Infosys)	INDIA
SatishKumar (Infosys)	INDIA
SatishKumar (Infosys)	INDIA
Ambuj Pathak (Infosys)	INDIA

Indian Classical Music - *Mousiqui* with Mukhi

"A Mathematician's Guide to Hindustani Classical Music": this pair of talks on the musical tradition of North India has been put together specially for the ICM. The first talk was delivered on August 22, 2010 and presented a brief history of Indian music, which has its roots in religious chanting from Vedic times around 5000 BCE. The textbook "Natya Shastra " by Bharata, the basis for the Bharata Natyam dance form presented at the ICM on Friday, has some reference to this music, and more details including an embryonic concept of raga appear in Matanga's Brihaddeshi in the 8th century.

By around the 11th century Persian and Arabic influences started to enrich the music and around this time the North and South Indian streams of music began to diverge. The present lectures focus exclusively on the North Indian or "Hindustani" tradition, which will be presented at the ICM in a live concert by Ustad Rashid Khan on Wednesday.

The nature of Hindustani music evolved during the 12th to 18th centuries, partly in response to the Bhakti movement in Hinduism, in which participatory and devotional love for the divine being (rather than formal worship of God as an idealised entity) became the principal theme. Another contributing factor was the patronage of the Mughal emperors. By the 18th century the "khayal" form of music was established. It remains an oral tradition even today, despite many books and treatises


Sunil Mukhi

on the subject, some of which have established a rudimentary notation.

In the first of these talks, the notion of raga is introduced by playing short clips of pairs of performances, different musicians, of the same raga. The common features between the members of a pair serve to illuminate the concept of the raga, even to a complete novice. A definition can then be built up through a series of successive approximations.

In its barest form, a raga is a set of notes selected from the 12 notes of the musical scale. But then these notes must be combined into patterns following certain rules. One can emulate the definition of a topological space in mathematics by saying that a raga R={S,U,T} is a subset S of notes from the musical scale together with a collection U of subsets of S and a set T of rules for combining elements of U! But art is not mathematics, so we need to add an aesthetics clause: the rules for combination must give rise to desirable results and create an appropriate mood. It is this mood that lies at the heart of a raga, which some authors consider to be a "living entity" rather than a mere combination of proportions and form. Parallel to raga, the concept of tala (rhythm) is briefly developed.

In the second talk the notion of "gharanas" or schools of music is briefly introduced (parallels with mathematics are quite strong!) and video clips used to illustrate some of the instruments and show how they are played. This is followed by a description of the structure of a typical performance, the different types of movements (introductory, slow and fast) and the complementary role of compositions and variations.

The bulk of the talk consists of audio and video clips of performances by some of the leading musicians of India (many of them sadly no more) illustrating different segments and features of a performance. In selected cases the lyrics and their significance are highlighted. The association of ragas with times of day and seasons is also briefly discussed. The talk closes with a short outline of the "lighter" forms: thumri, tappa and bhajan that are usually performed towards the end of a concert.

Mathematics and Music - A Divine Connection

Someone ignorant of music who looks at a piano keyboard would immediately notice certain properties: there are fewer black keys than white (five black to every seven white); the black keys are "clustered" in groups of two and three; and the pattern is "periodic" -- it repeats every 12 keys, or every 8 white keys. It is translationally invariant, as physicists would say.

We could try playing a melody starting from the white key (let's call it "C") that lies before a cluster of two black keys, and using only the white keys. Several simple melodies and nursery rhymes can easilybe reproduced in this way.

If we then try the same "pattern" of white notes, but start from the eighth white key (that lies before the next set of two black keys), the melody somehow sounds the "same" but at a higher pitch (just as a woman singing a melody sounds the "same" as a man, but at a higher pitch). So the translational invariance represents a musicalinvariance:ifthe first key is "C", the eighth should be called "C" too.

If we play a melody starting from the fifth key (if we name the whitekeys successively from "C", this key would be "G" and would lie after the first black key in a cluster of three black keys), again themelody sounds similar but no longer the "same". The pitch is not only higher, but different: yet it is clearly related. Also a certain note (the preceding white key, or the "F" note) sounds wrong.

This is understandable, because there is no black key before the C, but there is one black key before the G. If we just replace the white "F" note by that black key (call it F#), then we can play a melody starting on G in the same way we can with white keys starting on C; and there seems to be a connection, close but falling short of identity, between these notes and the notes that started on C.

These relationships are fundamental to all music and have been knownsince ancient times. The difference in pitch between one C and the next is called the "octave", and the difference between the C and the G (being the fifth white note counting from C) is called the "fifth".

In Indian music, if the C is the "sa" or "shadja", that is, the "tonic" note, the G is called the "pa" or "panchama" or, literally, fifth note. The fourth note, "F" or, in India, the "ma" or "madhyama" or "middle note", is also of significance: it is a fifth below the next "C" (or, if we started on F, the next C would be the fifth note).

A physicist may ask what the relationship is between these sounds,measured in terms of their frequencies. Again, the answer is ancient,but is obscured by the modern piano.

A thin vibrating string emits a frequency inversely proportional to its length (halving the length doubles the frequency). Pythagoras and the ancient Greeks knew that strings tuned to the same tension, whose lengths are in the ratios of small integers, emit notes that are in the ratios of small integers, emit notes that are "consonant" -- that is, sound good when played successively. One way to change the length of a string without changing its tension is simply to "stop" it with a hard object (which is the role of the "frets" on many

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A Key to Understanding Rhythm and Harmony

...Continued from page 2

on many stringed instruments). Halving the length of a string changes the pitch precisely by an octave, while changing its length by two-thirds (or pressing it at the 1/3 mark and striking the remainder) changes the pitch by a fifth. One can conclude (as did the ancient Greeks) that the octave corresponds to a doubling of frequency, while the fifth corresponds to a frequency increase of 3/2, or one and a half.

What about other small-integer ratios? It turns out that all of them are pleasing, and important, too: ratios like 4/3, 5/4, 6/5 all produce pitches that seem to "make sense" compared to the "base note" or "tonic". The smallest denominator that fails to appear in musical ratios is 7. In particular, notes in the frequency ratio 4:5:6 form what is called the "major triad": examples are C-E-G on the piano, sa-ga-pa in Indian music. These are the opening notes of the triumphant final movement of Beethoven's fifth symphony, of the refrain of the Beatles' "Ob-la-di, ob-la-da", and of many other well-known tunes. The chord that these notes constitute when played together, the "major chord", is the most important in Western music. In a sense, the reason for its importance is physical.

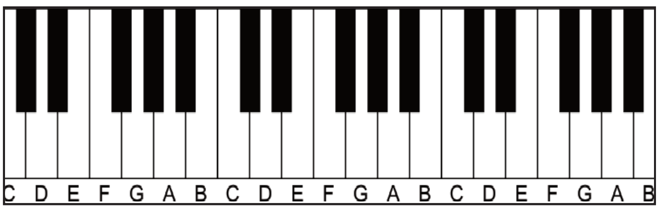
A string does not emit a "pure note" but includes "overtones" which are integer multiples of the fundamental frequency (as musicians know, these overtones can be selectively excited by gently "stopping" the string at nodal points while plucking it, but are always present anyway). For a string tuned to C, the first five overtones are 2, 3, 4, 5, and 6 times the fundamental frequency, corresponding to C, G, C, E and G respectively. So the C-E-G triad is (perhaps subliminally) present in the sound of a single string.

But there are many other keys on the piano: each octave contains twelve keys, seven white and five black. Why these twelve, and no others? The answer lies in a rather odd mathematical coincidence.

The white keys are named after the first seven letters of the alphabet: so the keys starting C are C, D, E, F, G, A, B. These are the notes of the "major scale" in Western music, and Indian musicians will recognise the scale as Shankarabharanam (Carnatic music) or Bilawal (Hindustani music).

Relative to C, the most important notes are G (whose frequency is 3/2 times C) and F (whose frequency is 4/3 times C: or, 3/2 times F is 2 times C, that is, the "next" C). What if we try to construct a major triad on each of these notes? (A pencil and paper may prove handy at this point, to follow the discussion.) With C, we get C-E-G = 4:5:6, so if C=1, we have E = 5/4 and G = 6/4 or 3/2. If we similarly start a major triad on G, we get G-B-D, so B = 3/2*5/4 = 15/8 and D = 3/2*6/4 = 18/8 (but we can lower this by an octave by dividing by 2, to get 9/8). And the major triad on F gives F-A-C, so if C = 1, we require A = 5/6 and F = 4/6. We can raise both these by an octave(multiply by 2) to get 5/3 and 4/3 respectively.

So we end up with seven notes with the frequencies C=1, D = 9/8, E = 5/4, F = 4/3, G = 3/2, A = 5/3, B = 15/8. These notes correspond to the "major scale" and are particularly pleasing when sounded successively, or together (it even seems obvious that D and B will sound the least pleasing when played together with C).



The keyboard of Western Piano

If we play these notes starting from the fifth, G, then one note -- F -- sounds "wrong". A higher note is required, which is the black key after F on the piano. If one starts in the next fifth note, D, then two black keys are required; and so on.

But those words "and so on" contain a subterfuge. Logically, the process should be never-ending: starting at any note, one can obtain a note that is a fifth higher. So, starting at C, we get a sequence that would be labelled by a musician as follows: C, G, D, A, E, B, F#, C#, G#, D#, A#, E#, B#, F##, ... Each note is obtained by multiplying the previous by 3/2, and if required, dividing by 2 to remain in the first "octave". The series never ends. To reproduce them all, a piano would need an infinite number of keys.

But the thirteenth note in the series, B#, is in fact very close to the starting note, C: if C is 1, B# is ((3/2)^12)/128 = 1.013+. This distinction is barely detectable to a good musical ear, and about a quarter of a "semitone", which is the smallest distance between two "white key" notes (that is, the difference between B and C).

So, some time in the seventeenth century, an inquiring mind asked: why not pretend that B# is the same as C? It would be the same if the "fifth" was defined to be, not 3/2 or 1.5, but 2^(7/12), which is about 1.498. The difference is just over one part in a thousand, imperceptible to most ears. And it gets even better: what if all musical intervals were powers of the twelfth root of 2, that is, the distance between any two adjacent keys on the piano was precisely 2^(1/12) = 1.059 roughly? It turns out that this gives a very good approximation to the nice "integer ratio" major scale above -- so good that most human ears can't tell the difference. And, since about the time of Bach (who was the earliest major composer to promote this system), all pianos have been tuned to "equal temperament". Of the first twelve powers of the twelfth root of two, seven are white keys and five are black keys in the first octave. Higher powers lie inhigher octaves.

This turned out to be very convenient for another reason. If we start a major scale on G, we noted and solved the problem with F, but there is a harder problem: the value we obtain for A is 3/2*9/8 = 27/16 or 1.69, which is (to musicians) audibly different (about a quarter-semitone) from the nice 5/3 or 1.67 that we obtained in the C major scale, but not so different as to merit a new key. With equal temperament, we get a compromise: A = 2^(9/12) = 1.68, just slightly different from the ideal value in both keys.

Equal temperament not only "closes the circle of fifths", as musicians put it, by making B# equal to C: it also allows musicians to "modulate" into different, and often quite remote, keys while playing the piano without sounding too out-of-tune in the process. The price for not being sometimes very out-of-tune is that one is always slightly out-of-tune: all notes are slightly (but, one hopes, imperceptibly) "wrong". The new flexibility engendered an immense and extraordinary diversity of Western music, from Bach through Mozart, Beethoven, Chopin, Schumann, Brahms, Wagner, Debussy, down to modern composers, jazz and rock music.

To conclude: Pythagoras and the Greeks, in noting the small-integer ratios of consonant sounds, saw a divine connection. (As Kronecker said two millennia later: "God made the integers; all else is the work of man.") However, since Bach's time, no intervals in Western music (except the octave) have been in integer ratios. All intervals are irrational. Metaphors for the modern world are left to the reader.

Rahul Siddharthan
Institute of Mathematical Sciences, Chennai.

A Musical Evening - Some Impressions

Sidharth Varma

In a rather relaxing and artistically engaging evening, the ICM delegates had an engrossing lecture on Indian classical music. The purpose of the lecture, which was given by Prof. Sunil Mukhi, a theoretical physicist from the Tata Institute of Fundamental Research (TIFR), Mumbai, was to initiate the audience into appreciating classical Indian music. It was the first part of the two part series.

With the aid of music clips and videos, the lecture captured the attention of the crowd as it explained the music. The lecture was narrowed down to Hindustani-Khayal form from the wide range of music that is available in the country. The talk was intended to be a primer to the uninitiated among the audience on how to listen to, and appreciate, the music, understand artist's creativity and the cultural underpinnings of this Indian classical art form.

Prof. Mukhi traced Hindustani music's origins to the 11th century and how the music evolved with adding of instruments such as the tabla (Indian drums) into the musical lineage from various countries. The well structured lecture gave definitions of raga (melody), tala (rhythm) and swara (notes). Prof. Mukhi also explained how these concepts in the Indian music were very different from the Western context.

Some short audio clips were played for the audience to enjoy and at the same time to decipher the similarity when played with different instruments. These included snippets from the performances of stalwarts like Ustad Vilayat Khan (sitar), Pandit Bhimsen Joshi (vocal), Ustad Bismillah Khan (shehanai) and Ustad Ulhas Kashalkar (vocal). Prof. Mukhi told the audience the things one should keep in mind while listening to a concert or a clip. He also advised the audience to try and listen to the music with eyes closed as this tended to heighten the sensory perception of Indian classical music. The ready availability of music clips on video streaming websites made it easier to source music and at the same time to be acquainted with them, Prof. Mukhi said.

The lecture-demonstration was a good initiation into Indian music ahead of Ustad Rashid Khan's Hindustani vocal recital on Wednesday, August 25, one of the cultural events associated with the ICM 2010.

INDIA and the IMU

Ever since its independence, India has been an enthusiastic participant in diverse international cooperative endeavours. The Indian National Science Academy was identified early on by the Government of India as the agency through which Indian participation in international scientific bodies was to be implemented. The Acdemy formed National Committees in different disciplines to be adhering bodies of international scientific unions. In 1950 when the distinguished American mathematician Marshall H Stone initiated efforts to revive the Interantional Mathematical Union (IMU) which had become defunct a few years before the start of the Second World War (largely because of Franco - German tensions), the Indian National Committee took a lot of interest in the writing of a new constitution and formulation of the statutes. Professor Stone in an official statement thanks the Indian National Committee for its help in framing the constitution and the statutes. The Indian mathematician who played a key role in the run-up to the formation is K Chandrasekharan. Chandrasekharan turns 90 this year.

Chandrasekharan served on the IMU Executive commitee for 24 years srating 1954, the year in which India became a member of the IMU. During 1970 - 74 he was President of the IMU and thus the chair of the Fields Medal Committee. Earlier in his tenure he took on the responsibilty of publishing the World Directory of Mathematicians which listed names of mathematicians from all the member countries of the IMU: the criterion for being listed in the Directory was that the person have two papers reviewed in the Mathematical Reviews or the Zentralblatt fur Mathematik.

Since 1978 except during the periods 1978 - 86 and 2006 - 10, there has always been an Indian mathematician on the IMU Executive commitee (EC). V Srinivas of Tata Institute of Fundamental Research has been elected (at the general assembly in Bangalore) a member of the EC for 2010 - 14. Indian Mathematicians have also served on other committees of the IMU.MS Narasimahan and S G Dani have been chairs of the Commision on Develoment and Exchanges (CDE). The new avatar of the CDE which goes by the name "Commission for Developing Countries" (CDC) and S Kesavan has been elected to serve as CDC Secretary Grants.

Chandrasekharan was responsible for several inititiatives taken by the IMU. One of them was the sponsoring a quadrennial conference called Internatioanl Colloquium in Mumbai: these have ben held continuously from 1956 on with substantial support from the IMU except in 1976.

Mechanisms for strengthening mathematics in developing countries

The London Mathematical Society (LMS) will hold a Discussion Meeting on Wednesday, August 25, 2010, during 1700-1900 hrs in Hall 2 of the Hyderabad International Convention Centre (HICC), the venue of the ICM. The meeting will highlight three mechanisms for strengthening mathematics in developing countries: the Mentoring African Research in Mathematics project (MARM), the IMU Volunteer Lecturer Program, and the work of the International Centre for Theoretical Physics (ICTP) Trieste.

In the MARM project the mentor, a mathematician from a developed country, works closely with a research group in a mathematics department in Africa, providing detailed support and advice. MARM is four years old and has set up thirteen mentoring partnerships.

In the IMU Volunteer Lecturer Programme, intensive 3–4 week courses in mathematics at the advanced undergraduate or master's level are offered to universities in developing countries.

ICTP has a long tradition of helping mathematics in developing countries using a wide variety of different approaches including visiting positions, support for networks, and the Associateships Scheme.

Questions to be addressed at the meeting will include: What are the best mechanisms? What principles should underlie them? Are there effective ways in which different types of project can cooperate? How can individuals and institutions contribute?
(isabelle.robinson@lms.ac.uk)

On 24th August 2010, World Chess Champion Viswanathan Anand will play simultaneous Chess with 35 mathematicians and five nominees of Infosys which has been very generous in its support to ICM 2010. The youngest player is Palden Ball who is just 12!

Announcement

Anand versus 40
Simultaneous Chess
*at **15:00, Hall 2***

Special lecture by Fields Medalist S. Smirnov
13:45-14:45**, **Hall 4

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