ICM 2010
HYDERABAD • INDIA

REFLEXIONS

The daily newsletter of ICM 2010

August 25, 2010


Michel the Missionary 2


Anand and 2560 Squares 3


Concert: Ustad Rashid Khan 4

Sound Approach to Quantum Unique Ergodicity

Photo :Rahul V Pisharody

Kannan Soundararajan

Kannan Soundararajan is a Professor of Mathematics at Stanford University. He works in number theory, especially L-functions and multiplicative number theory. He won the Morgan Prize in 1995 for his work in analytic number theory while still an undergraduate at the University of Michigan. He also won the Salem Prize (with Lindenstrauss in 2003) and the SASTRA Ramanujan Prize (shared with Manjul Bhargava in 2005).

When he was an undergraduate student at the University of Michigan, Soundararajan proved two outstanding results. Firstly, he proved a conjecture of Graham in combinatorial number theory in collaboration with R. Balasubramanian. Then, he proved some deep results on the distribution of zeros of the Riemann zeta function.

In his PhD thesis, Soundararajan proved that more than 7/8-ths of quadratic Dirichlet L-functions have no zeros at the critical point s=1/2. He proved with Ken Ono, a conjecture of Ramanujan under Generalized Riemann Hypothesis (GRH). Soundararajan is an expert on random matrix theory where he showed with Hugh Montgomery that the distribution of prime numbers in short intervals is surprisingly different from what classical heuristics imply. Soundararajan, considered to be one of the most creative young minds to emerge in the last decade.On August 22, he delivered a talk at the ICM 2010 on `Quantum Unique Ergodicity and Number Theory'.

The Riemann zeta function encodes many properties about prime numbers. The prime number theorem which addresses the asymptotics of the number of primes, was conjectured by the young Gauss in the late eighteenth century. It was proved about a century later independently by Hadamard, and de La Vallee Poussin. One expects to have good estimates on error terms too which we do not yet have. This is related to the famous Riemann Hypothesis which is one of the Clay Prize problems.

There are many conjectures concerning the values of zeta and other L-functions on the critical line - the line of symmetry for their functional equations - the Riemann Hypothesis and Lindelof Conjecture, for instance. Among them there are conjectures about the growth of these L-functions on the critical line. There are some simple estimates, called the convexity bounds about the values of these L-functions on the critical line. In several questions, `breaking' this convexity bound is of paramount importance, and is known for a class of L-functions, and in each case it is rather nontrivial.

Soundararajan has achieved such a theorem for what is known as the triple product L-functions, and as a result proved what has been called the Quantum Unique Ergodicity (QUE) conjecture of Rudnick and Sarnak.

The QUE conjecture asserts that the so-called newforms, which are holomorphic modular forms on the Poincare upper half plane, and are eigenfunctions of Hecke operators, give rise to measures on the upper half plane which converges to the invariant measure on the upper half plane as you vary either the weight of the modular form keeping the level fixed, or vary the level keeping the weight fixed. What has been particularly spectacular about Soundarajan's work is that if it is combined with a parallel develpoment in the subject by Roman Holowinsky, it yields marvellous results. They each prove theorems towards QUE, neither settling it completely, but in the end one realizes that together they have completely proven the QUE conjecture!

In this context, one should note that Elon Lindenstrauss proved the QUE when one considers Maass forms instead of holomorphic modular forms. This is part of the work cited while awarding Lindenstrauss the Fields medal. Incidentally, Soundararajan and Lindenstrauss won the Salem prize in the same year.

Soundararajan's work is considered by the experts in the subject as one of the deepest contributions to the subject in the last few years. Given the interest in this subject, it is in a state of flux, with many mathematicians trying their hand at simplifying and unifying the arguments.

R.Balasubramanian **on Soundararajan, Page 3**

"The result of Holowinsky and Soundararajan comes from their separate attacks on the holomorphic QUE problem and depends on the Ramanujan conjecture proved for certain holomorphic forms by P. Deligne in the 1970's. Each method achieves the desired result up to a small number of exceptions. Remarkably their approaches are sufficiently different so that the set of exceptions to both approaches, is empty! "

-Peter Sarnak, Princeton University

"A part of it is complicated; that's the beautiful thing, in fact. Our approaches [to the problem of QUE] are completely different. Actually I got some help from him many years ago His approach is purely number theoretic. He does not use any dynamics. The difficulties I have are different from the difficulties he has in his approach. It's one of the beauties of the problem. One of the things I am trying to think about using some ingredients from his and combine with my kind of techniques. It will be quite an interesting direction. "

- Elon Bruno Lindenstrauss

"One of the things that Elon Lindenstrauss got his Fields Medal this time for is settling a case of the Quantum Unique Ergodicity (QUE) Conjecture. He settled it for some objects which are called Maass forms. These are certain non-holomorphic forms on the space. There is also the analogous case of holomorphic forms which is not addressed by his theory. They are kind of two sides of the problem but the methods are completely different. Our approach is number theoretic. Elon's approach comes from dynamical systems and ergodic theory. "

-K.Soundararajan

Mathematical Visits to Developing Countries - A Passion

Photo: Rahul V Pisharody

Prof. Michel Waldschmidt

Michel Waldschmidt is a Professor at the Faculté de Mathématiques Pierre et Marie Curie. He works in Transcendental Number Theory. Waldschmidt has been actively involved in the mathematics cooperation between France and several countries in Asia, Africa and South America. He is passionately interested in running and mountaineering. Let us hear what he says when he talks to **Richa Malhotra and B. Sury**

You have been actively associated for several years with cooperations in several countries including India. What inspires you to involve yourself with these activities?

From a very young age, I wanted to teach mathematics in developing countries and my first opportunity was in 1976 when I was invited by Prof. K. Ramachandra. I visited the Tata Institute for 3 months and this was a very tremendous and impressive experience for me. I was not speaking good English at that time; so I also learnt English by being with Prof. Ramachandra.

This was an experience which was important for me. After that, I felt that I needed to be more effective; I was picked by my Government for that. I think what I do is more useful - helping students in developing countries to do mathematics. When I teach in a place like India, in many places, I meet a lot of students who are very eager to learn much more than in France. When I go to places like Bhubaneswar (India), Abdus Salam Institute (Lahore), I meet students who are very enthusiastic about mathematics and ask a number of questions. This is what I like. For me teaching is a pleasure.

You have a passion for running and mountain climbing. So what is the extent to which you have gone into these things?

I was running since I was young. It always gave me great pleasure. When I was running I was forgetting mathematics; so, for me it was good to have two activities which were completely different. In order to do mathematics, we need to do something else also. But, the extent I went to is too far because I wanted to run a marathon. I have run now more than 20 marathons and, now I cannot improve my speed. So, last year I went to 100 kilometres and was for 24 hours.

I am hesitant to ask this question. Did personal losses like passing away of your daughter in 2004, motivate you to do more for the deprived sections of society? How do you find outlet of emotions leading from such an event?

It is always something which is emotional for me.I have put some information on my web page and this means I am willing to speak about this.

People react differently, for example my wife does not like to speak on that and she often says she wants to forget but it is not my case, I don't want to forget. Yesterday I met a colleague who spoke of my daughter because he saw my web page, and I told him that when I speak of her, it was as if she were here. But, I like to speak of my daughter although she doesn't exist. Whatever I do, I wish that she would be proud of me; this has a lot of influence in that way.

You instituted a foundation as a tribute to your daughter. Would you like to highlight about the activities of this foundation?

Yes, well I will tell you the truth, it is not working very easily. After she passed away some people gave us some money, which usually in our culture is used for flowers. But we thought that it would be a waste, so we said we would like to use it for Mali because one year before she passed away when she was in Mali in a training session in a hospital, we thought we would send the money to the hospital and then donate it to the medical centre which was just created at the same time.

In fact we got a lot of money but it was very difficult to use this money. But, to help people, for example, when they want to build some building, we say we are ready to give you the money but you have to give the plan and get the authorization of the government. They do not do that and we have to insist. We have now found one centre which should work in another hospital. For me it was an experience to see how difficult it is when we want to help people. It is a part of my life. In mathematics, we are doing work in Africa; things are quite different there.

Where do you see the future of mathematics heading in the near future? Do you see it getting related to computer science or both?

I read this text by Terrence Tao. He explained very well the future of mathematics and it is explained so well that it is difficult for me to say something after he said it so beautifully. It is really very difficult to know what to say; it depends on whether you talk of the near future or in the long term. Tao has spoken about whether computers will replace our activity as mathematicians. My field is number theory – in particular, Diophantine equations. There is Hilbert's 10th problem which says in a sense that we can never find a complete theory for all Diophantine equations.

It is good for mathematicians because it is our experience that whenever we close one problem, we may create a new field or whenever we prove a new theorem we ask some more questions. This is good for the future and we expect that the number of problems will be large. In my career I saw very big problems which are solved now. I think that in the near future there will be tremendous results which solve what we consider as difficult now. Some new ideas will be needed and when these will come, nobody can tell. When we are asked to research we can't say next year I will prove this and in two years I will do this.

Do you feel the need to motivate yourself to do mathematics research?

This is a personal question in the sense that people react differently. For me it is important because it motivates the research subject when I work on an open problem. I ask myself if it was worth spending much time on it. If I spend one year or two years, it may not be worth it unless I can see some applications. It may be worth spending just one or two months. But, if I spend all this time to solve the problem and nobody is interested (including myself!), it is not worth it. I need to know that the problem is motivating in the sense that it has some applications.

My motivation is inside mathematics, but that is sufficient. I would not work on the problem if I don't see that this problem is interesting for me or for other questions.

...continued on page 4

'I am really proud to have played some role in his formative years'

Kannan Soundararajan or Sound as he is affectionately called met me in 1986/87, when he was just finishing middle school and entering high school. When he first started visiting me at Matscience, the Institute would have a regular working day on Saturdays, and he could come then without missing school. Some time later, Saturdays became holidays, but I invited him then to come to my house on Saturdays, and we would then discuss mathematics. I can still recall the days when he used to come to meet me.

I remember, one Friday evening when he met me while I was looking at apaper of Alladi, Erdos and Valer on the summation of arithmetic functions. I suggested to Sound to study the paper in the weekend. Two days later on Monday morning, he was waiting for me in the office with a simple proof of an improvement of this result. This later appeared in the *Journal of Number Theory*. While still in high school, he spent a month at TIFR Bombay to attend a course of lectures given by Alexander Ivic on Riemann zeta function. This kindled his interest in zeta function and we used to discuss a lot on this.

He went to the United States for a six week summer programme called the Research Science Institute (RSI), which used to be held in Washington DC. It may be interesting to note that the other students working with Lawrence Washington then were Terrence Tao and Lenny Ng. Washington suggested to Soundararajan a problem on the number of primes in certain sequences. Sound solved the problem using Selberg sieves and the paper appeared in *Journal of Number Theory*. Another problem which I suggested to him was a conjecture of Graham on the greatest common divisor (gcd) and at that time Zaharescu had just proved the conjecture for all large values. Sound came up with some beautiful ideas which unfortunately did not lead us anywhere. Then I lost interest in the problem but Sound persisted with the problem and got a complete solution in three years.

When I was visiting him in Michigan for a couple of days , we settled a conjecture of Erdos on distinct divisors and submitted it to a journal. When the referee pointed out that our method is likely to yield stronger results, Sound was not ready to publish the paper in that form but went far beyond Erdos's Conjecture.

R.Balasubramanian

I am really proud to have played some role in the formative years of a mathematician who rose to great heights proving results on diverse topics like Quantum unique ergodicity, upper bound for the mean values of zeta function on the critical line, structure of multiplicative functions etc.

Viswanathan Anand and 2560 Squares!

Photo : Rahul V Pisharody

Vishy Anand playing simultaneous chess with delegates at the ICM 2010.

Richa Malhotra

Viswanathan Anand played chess against 40 people simultaneously on 24 August 2010. All lost except for a 14-year whiz kid who drew with him. This is Srikar Varadaraj from Bangalore who also has the distinction of being one of the youngest to present a paper at the ICM.

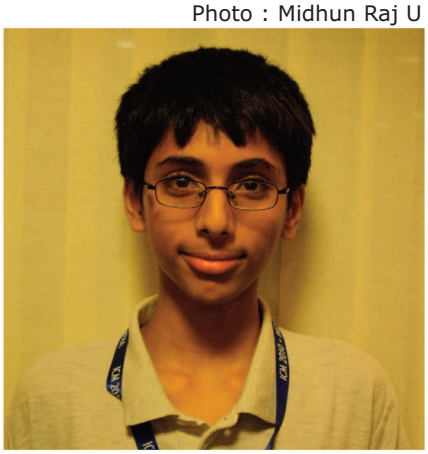
Srikar says he wants to be a mathematician after growing up. He was among the top 30 selected from Karnataka to participate in the Indian National Mathematical Olympiad in 2009. About the game, Srikar later said, "I traded the Queen and thought I will catch up in the end game, but Anand is too good at end games!" Srikar's father was indeed glad to see his son perform neck-to-neck with Vishy Anand.

Srikar studies in Venkat International Public School, Bangalore, and likes listening to rap music, playing chess and solving Mathematics Olympiad problems. While Anand learnt chess from his mother at the age of six, the 14-year old Srikar learnt it from his father.

About Srikar, Viswanathan Anand said, "Oh! He was great". Anand felt that the level of the game was extremely high and the last 10 players held on very tenaciously. He said, "Playing with the participants here was more of a rollercoaster ride while playing against one person is controlled". He adds, "I was completely busted. A lot of people here played very well." During the question and answer round, he exclaimed that there are periods when one does well and when one does badly. Anand drew a correlation of chess with mathematics and remarked, "Mathematics is similar to chess. If kids get fascinated in mathematics early, it stays for long just like in chess. Chess is one thing you can teach kids very early. Some countries and communities tend to expose them to chess more. But we need more information to prove this." The game was drawn after 46 moves.

Anand played with white pieces, while the others played with black.

Photo : Midhun Raj U



Srikar Varadaraj

'Mathematics is growing at a speed which is exponential'

...continued from page 2

I work on it only if I see this problem is inside some theory. It makes some sense to people - this is the kind of motivation that I want.

Who inspired you to pursue mathematics?
Well, among the great mathematicians, one is Serre. He is a Fields medallist. The way he does mathematics is very impressive; the way he writes mathematics is great. He is a very nice man also, I did not anticipate this question and I did not know the answer before you asked. This is the first name I have, Wolfgang Schmidt. He proved something which is called the subspace theorem, which is one of the most important results of the twentieth century. Now the number of applications of the subspace theorem is large.

Does professional rivalry have a negative impact on the development of the subject of mathematics or is it sometimes good also?
When I was a student, and when we write a dissertation, we are told to look at the positive side as well as the negative side and so, I will do that. On the negative side, I often meet people who say they are working on a particular problem and don't want to tell others what they are doing because someone could steal their work. I think this is very bad for the development of mathematics.

I would think of mathematicians all working together to fight against conjectures and to convert conjectures into theorems. That should be done together and not against one another. Whenever I hear the question of priority, I think it is wrong. People should not insist that they had the idea first as it is bad for the development of mathematics. On the other hand, I am also a runner and I like competition. I think competition can be good; it can be a challenge but it should not be a challenge of one against the other. On the whole, what I think is that to work together should be encouraged.

''I would consider that mathematicians come together to fight against conjectures and to make conjectures into theorems. That should be done together and not against one another.''

.

What are the things you would like to be changed in mathematics?
Diversity is something which should be preserved. We should have some great mathematicians and also some mathematicians who do not make great researchers but good teachers.

In a country like India, there is space for more mathematicians. In a global setting, there should be more support for mathematics because, as a subject, mathematics is not so expensive to pursue. It is not true that mathematics needs only paper and pencil, as people say. This is not completely correct; they need more. For example, one needs computers, books and journals and one needs funding for travel. But, this still costs less than what the expenses are for pursuing experimental physics or biology or chemistry. It is true that if funds for mathematics are increased by a little, this would have tremendous consequences. To convince the government is something which is usually difficult. But I think in India, you have a strong support from the government. This is something extremely good and very important for the development of mathematics in India. The fact that the ICM is taking place in India is a proof of that. Also, we have the NBHM which is doing such a good job and they deserve to get more support.

Do you think mathematicians of the last century did deeper work than those in the previos century, in the sense that big conjectures got solved now than ever before?
It is a continuous process. When you look at the history and development of mathematics, the development is growing quite a lot. The mathematics which was done in the previous centuries is not destroyed and we use the whole of what was done then. I think lot of things have been done, in the last century, especially in the last half century from 1950 to 2000. It's growing at a speed which is exponential and it is really amazing to see.

How do you think we can enhance the communication of mathematics to the public?
This is a very important subject. This is something in France which was not good. A good mathematician would not explain mathematics to a layman, it was not his job. It has completely changed now. Many good mathematicians give some popular lectures, for e.g. few months ago I attended a public talk by Villani meant for students of high school. When I was the President of the French Mathematical Society we organised a meeting for the public, titled "The hidden face of mathematics". Just like the hidden face of the moon, there is a part of mathematics that you don't see.

Use of Metric In Evaluating Research

Panellists: László Lovász (Chair), Douglas Arnold, José-Antonio de la Peña, Malcolm MacCallum,Frank Pacard.

The use of metrics for evaluating research is a hotly debated issue. The IMU/ICIAM/IMS report on Citation Statistics1 highlighted the dangers of uncritical use of impact factors, which play an increasing role in funding, promotions and library purchases. Are impact factors and other such indices good measures of journal quality, and should they be used to evaluate research andindividuals? What can be done about unethical practices like impact factor manipulation? Is there a role for metrics in evaluating research? Are there better alternatives?

The Citation Statistics report can be found at
http://www.mathnion.org/fileadmin/IMU/Report/CitationStatistics.pdf

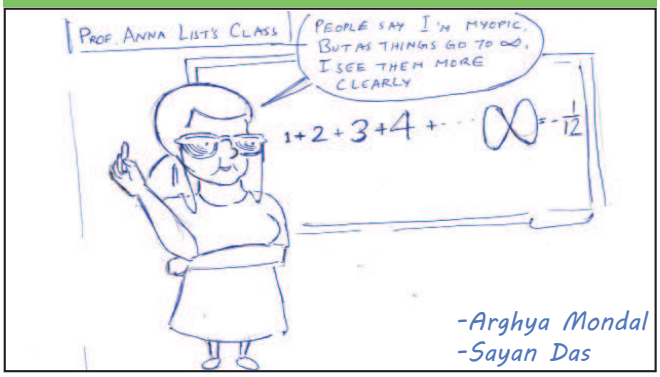
Panel Discussion

Communicating mathematics to society at large

Chair: G. M. Ziegler, Technische Universität, Berlin, Germany

15:00 - 17:00, Hall 2

Mathaloon



-Arghya Mondal
-Sayan Das

Announcement

Ustad Rashid Khan (Vocal) Hindustani Classical Music Concert at ***19:00, Hall 2***

Simon Singh Popular lecture, ***17:00 - 18:00 Hall 4***

Newsletter Team

R.Ramachandran
B.Sury
Geethanjali Monto
Richa Malhotra
Midhun Raj U.R
Mohammed Anvar T
Rahul V Pisharody
Sidharth Varma
Nikhil MG