Citation for Chern medal award to Masaki Kashiwara, 2018

Short sentence

The Chern medal is awarded to Masaki Kashiwara for his outstanding and foundational contributions to algebraic analysis and representation theory sustained over a period of almost 50 years.

Longer paragraph

Masaki Kashiwara's mark on current mathematics is exceptional. It extends from microlocal analysis, representation theory and combinatorics to homological algebra, algebraic geometry, symplectic geometry and integrable systems. Most notable are his decisive contributions to theory of D-modules and his creation of crystal bases, which have shaped modern representation theory.

Over a span of almost 50 years he has established the theory and applications of algebraic analysis. Introduced by Sato around 1960, algebraic analysis is a framework in which systems of linear differential equations are formulated as modules over a ring D of differential operators and are analysed with algebraic means such as rings, modules, sheaves and categories. Sato's idea of D-modules was greatly developed by Kashiwara in his 1971 thesis, and has become a fundamental tool in many branches of mathematics. With Sato and Kawai, he developed microlocal analysis, the study of partial differential equations, locally combining space and Fourier variables, and working on the cotangent bundle of the base manifold. In the 1980s with Schapira he further introduced and developed microlocal sheaf theory.

One of his early major results was his 1980 construction of the Riemann-Hilbert correspondence, a far-reaching generalization of Hilbert's 21st problem about the existence of a linear differential equation on the projective line with prescribed monodromy. Greatly generalising work of Deligne, Kashiwara (and later independently Mebkhout) established an equivalence between the derived category of regular holonomic D-modules on a complex algebraic variety ("systems of linear differential equations") and the derived category of constructible sheaves on the same variety ("solutions"), thereby creating a fundamental bridge between algebra and topology.

The Riemann-Hilbert correspondence found a remarkable application to a problem in representation theory: the Kazhdan-Lusztig conjecture, proved independently by Brylinski- Kashiwara and Beilinson-Bernstein. This beautiful synthesis of algebra, analysis and geometry may be viewed as a precursor to geometric representation theory in its modern form. With Tanisaki, Kashiwara generalized the conjecture yet further to infinite- dimensional Kac-Moody Lie algebras, a critical ingredient for the completion of Lusztig's program in positive characteristic.

Kashiwara's 1990 development of the theory of crystal bases is another landmark in representation theory. Quantum groups are deformations of the enveloping algebras of Kac-Moody Lie algebras originating from certain lattice models in statistical mechanics. Here the deformation parameter q is temperature, with q = 0 corresponding to absolute zero. Kashiwara’s crystal bases are bases of representations of quantum groups at q=0 equipped with certain graph structure, which encode essential information about the representations. Being a powerful combinatorial tool, crystal bases have had great impact with many applications, including solving the classical problem of decomposing tensor products of representations. In 1991, Kashiwara showed that for all q, they lift uniquely to global bases. These later turned out to coincide with the canonical bases discovered by Lusztig in 1990, for both the q=0 and generic q cases, from a totally different viewpoint.

Kashiwara's work, with many different coauthors, continues to be groundbreaking. He has been at the forefront of recent developments on crystal bases, including the categorification of representations of quantized enveloping algebras. Other highlights include the solution of two well-known open problems about D-modules: the `codimension 3 conjecture' (2014) and the extension of the Riemann-Hilbert correspondence to the irregular case (2016). His work on sheaf quantization of Hamiltonian isotopies (2012) opened the way to applications of microlocal sheaf theory to symplectic geometry.

Kashiwara has many other results too numerous to mention. For example, he obtained major results on representations of real Lie groups, and on infinite-dimensional Lie algebras, in particular in connection with integrable systems and the KP hierarchy.

Kashiwara's influence extends far beyond his published work. Many of his informal talks have initiated important subjects and his ideas have been a source of inspiration for many people. Several of his books have become essential references, his book on sheaves with Schapira being regarded as the bible of the subject. He has served as Director of RIMS Kyoto and was Vice President of the IMU 2003-2006.

Kashiwara's work stands out in depth, breadth, technical brilliance and extraordinary originality. It is impossible to imagine either algebraic analysis or representation theory without his contributions.