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Abstracts of Plenary and Invited Lectures

Section:

0. Plenary Lectures

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Mathematical problems in geophysical wave propagation

The analysis of wave propagation in randomly inhomogeneous media leads to many difficult problems in stochastic processes and differential equations. Wave propagation in the earth's crust, seismic wave propagation, is particularly difficult to model, to analyze, to simulate and to use as a probing tool. In this lecture we will present a review of the mathematical problems that arise in seismic wave propagation and the role that modern applied mathematics can play in articulating them and in providing methods and tools for their solution.
After a brief review of some basic notions in waves in random media, such as the identification of the relevant length and time scales and properties of the coherent wave, we will describe the mathematical modeling of energy propagation in two important regimes:

- **Long wave, long distance wave propagation and the use of Radiative Transport Theory**
- **Local, exploration seismology, anisotropy and wave localization**

The main issues in the use of Radiative Transport Theory in seismic wave propagation are the following.

- When is it a valid description of the observed phenomena?
- When can it be replaced by a much simpler Diffusion Theory?
- What do we know about reflection and transmission at interfaces or boundaries when Radiative Transport is valid?
- What do we know about wave mode energy conversion in the Radiative Transport regime?

These questions have precise mathematical formulations and considerable progress in their analysis has been made in the last five to ten years. But many important problems remain open. The behavior of random waves near boundaries and interfaces is particularly difficult to analyze. New analytical methods from microlocal and stochastic analysis are needed for the more basic questions. Even methodologically simpler problems, like the passage of Radiative Transport to Diffusion, are difficult to analyze near boundaries and many problems there are still open. It is interesting to note that the use of Radiative Transport in seismology is relatively recent, mostly in the last fifteen years, while Radiative Transport has been used to model the passage of light through inhomogeneous atmospheres for nearly a century.

The earth's crust is, however, a very complex medium and Radiative Transport need not be a suitable analytical tool when the inhomogeneities are strong and highly anisotropic. This is the case frequently in exploration seismology where propagation is confined to a few kilometers and wavelengths are relatively short (about one hundred meters). We are now close to the regime of **Wave Localization** where the random inhomogeneities act to limit spatially the propagation of wave energy. We know a lot about the mathematics of Wave Localization in randomly layered media and we will review briefly several results obtained in the last few years. But there are many difficult open problems when it comes to understanding the localization-delocalization (or transport) transition as we go from perfectly layered random media to highly anisotropic ones and then to more isotropic random inhomogeneities. We know very little here and numerical simulations do not help because the kind of resolution required to observe localization phenomena is impossible to achieve at present.

We will conclude the lecture with a brief review of computational issues, parameter estimation and imaging in the presence of random inhomogeneities.